Time: 3 Hours **MATHS TEST 3** Max. Marks: 100

General Instructions:

(1) *All questions are compulsory.*

(2) *The Question Paper consists of 29 questions divided into 3 Sections A, B and C. Section A comprises of ten questions of 1 mark each, Section B comprises of twelve questions of 4 marks each and Section C comprises of seven questions of 6 marks each*

(3) *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*

(4) *There is no overall choice. However, internal choice has been provided in four question of 4 marks each, two questions of 6 marks each You have to attempt only one of the alternatives in all such questions.*

(5) *Use of calculators is not permitted. However, you may ask for mathematical tables.*

SECTION – A

Q1. If $f : R \to R$ be given by $f(x) = (3 - x^3)^{\frac{1}{3}}$, then find $fof(x)$.

Q2. Find AB, if $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$.

Q3. If $\vec{a}, \vec{b}, \vec{c}$ are non zero vectors and $\vec{c}$ is perpendicular to $\vec{a}$ & $\vec{b}$. Show that $\vec{c}$ is perpendicular to $\vec{a} - \vec{b}$.

Q4. Find values of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

Q5. Find the values of : $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

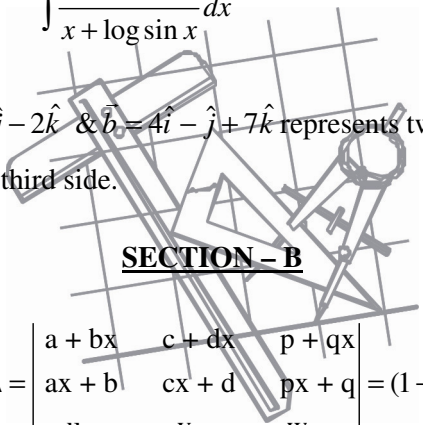
Q6. Evaluate : $\int_0^1 xe^x dx$

Q7. If $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ & $\vec{b} = 4\hat{i} - 4\hat{j} + 7\hat{k}$, find the projection $\vec{b}$ of on $\vec{a}$.

Q8. Find the values of x, y, z if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfy the equation $A'A = I$.

Q9. Evaluate : $\int \frac{1+\cot x}{x+\log \sin x} dx$

Q10. If $\vec{a} = 3\hat{i} + 6\hat{j} - 2\hat{k}$ & $\vec{b} = 4\hat{i} - \hat{j} + 7\hat{k}$ represents two sides of a triangle, then find the third side.



SECTION – B

Q11. Prove that $\Delta = \begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = (1-x^2)\begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$

Q12. Evaluate : $\int \frac{x^2}{x^4+1} dx$

OR

Evaluate : $\int \frac{x+2}{(x+1)(x^2+1)} dx$

Q13. Find p, if the points (1,1,p) and (-3, 0 , 1) are equidistant from the plane whose equation is $\vec{r}.\left(3\hat{i}+4\hat{j}-2\hat{k}\right)+13=0$.

Q14. Find the value of k for which the function is continuous

$$f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x}, & \text{if x } \neq \dfrac{\pi}{2} \\ 3, & \text{if x} = \dfrac{\pi}{2}\end{cases} \qquad \text{at x} = \frac{\pi}{2}$$

Q15. Evaluate : $\int \frac{x+3}{\sqrt{5-4x-x^2}}dx$

Q16. A discrete random variable X has mean 6 and variance 4. Assuming that the distribution is binomial, find is probability function and obtain. (i) $P(3 \le x \le 5)$ (ii) $P(x \ge 3)$

Q17. Prove that $y=\frac{4\sin\theta}{(2+\cos\theta)}-\theta$ is an increasing function of θ in $\left[0,\frac{\pi}{2}\right]$

Q18. Let X be a non-empty set P(x) be its power set. Let '*' be an operation defined on elements of P(x) by, Then,

(i) Prove that * is a binary operation in P(X).

(ii) Is * commutative ?

(iii) Is * associative ?

(iv) Find the identify element in P(X) w.r.t. *

(v) Find all the invertible elements in P(X).

(vi) If o is another binary operation defined on P(X) as A o B = then verify that o distributes itself over *.

Q19. Differentiate $\cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]$ w.r.t. $\tan^{-1}\left[\frac{3x-x^3}{1-3x^2}\right]$.

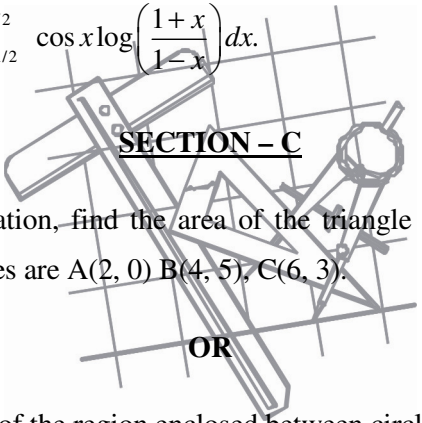
Q20 Show that $\sin^{-1}\frac{3}{5}-\sin^{-1}\frac{8}{17}=\cos^{-1}\frac{84}{85}$

OR

Simplify $\tan^{-1}\left[\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right]$, if $\frac{a}{b}\tan x>-1$.

Q21. Find the vector equation of the line parallel to the line $\frac{x-1}{5}=\frac{3-y}{2}=\frac{z+1}{4}$ and passing through the point(3, 0, – 4). Also find the distance between these two lines.

Q22. Evaluate : $\int_{-1/2}^{1/2}\cos x\log\left(\frac{1+x}{1-x}\right)dx.$



SECTION – C

Q23. Using integration, find the area of the triangle ABC, coordinates of whose vertices are A(2, 0) B(4, 5), C(6, 3).

OR

Calculate area of the region enclosed between circles

$\left(x-\frac{1}{2}\right)^2+y^2=1$ & $x^2+y^2=1$.

Q24. Show that the semi-vertical angle to the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

Q25. For the matrix $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$. Show that $A^3-6A^2+5A+11I=O.$

Hence, find A^{-1}.

Q26. Find the distance of the point (3, 4, 5) from the plane x + y + z = 2 measured parallel to the line 2x = y = z.

Q27. Two groups are competing for the positions on Board of Directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, it the first group wins, the probability of introducing a new product is 0.7 and corresponding probability is 0.3 if second group wins. Find the probability that the new product introduced was by second group.

Q28. (i) Solve the following differential equation :

$(x - y^3)dy + y\ dx = 0.$

(ii) Show that the differential equation of which

$1 + 8y^2 \tan x = c\ y^2$ is a solution, is $\cos^2 x \frac{dy}{dx} = 4y^2$.

Q29. A farmer decides to plant up to 10 hectares with cabbages and potatoes. He decides to grow at least two but not more than 8 hectares of cabbages and at least 1 but not more than 6 hectares of potatoes. If he can make a profit of Rs.1500 per hectare on cabbages and Rs.2000 per hectare on potatoes, how should he plan his forming so as to get the maximum profit ?

OR

A dietician wishes to mix two types of food in such a way that vitamin content of the mixture contain at least 8 units of vitamin A and 10 units of vitamin B. Food I contain, 2 units/kg. of vitamin A and 1 unit/kg. of vitamin B while food II contains 1 unit/kg of vitamin A and 2 unit/kg of vitamin B. It costs Rs.5 kg. to purchase food I and Rs.7/- kg to purchase food II. Determine the minimum cost of such a mixture.