Time: 3 Hours **MATHS TEST 1** Max. Marks: 100

General Instructions:

(1) All questions are compulsory.

(2) The Question Paper consists of 29 questions divided into 3 Sections A, B and C. Section A comprises of ten questions of 1 mark each, Section B comprises of twelve questions of 4 marks each and Section C comprises of seven questions of 6 marks each

(3) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(4) There is no overall choice. However, internal choice has been provided in four question of 4 marks each, two questions of 6 marks each You have to attempt only one of the alternatives in all such questions.

(5) Use of calculators is not permitted. However, you may ask for mathematical tables.

SECTION – A

Q1. If $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{bmatrix}$ and B = $\begin{bmatrix}3 & -1 & 3\\ -1 & 0 & 2\end{bmatrix}$, then find 2A – B.

Q2. Evaluate : $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

Q3. Find the number of all onto functions from the set {1, 2, 3,…..,n} to itself.

Q4. Evaluate : $\int\frac{xe^x}{(x+1)^2}dx.$

Q5. If A=$\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$, then find the value of α if $A+A'=I$.

Q6. Find the value of p if $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+3\hat{j}+p\hat{k})=\vec{0}$

Q7. Write a value of $\int\frac{4}{4+x^2}dx$

Q8. Write the distance of the following plane from origin : $2x-y+2z+1=0$

Q9. Find the angle between two vectors $\vec{a}$ & $\vec{b}$ if $|\vec{a}|=3, |\vec{b}|=4$ & $|\vec{a}\times\vec{b}|=6$.

Q10. Assume X, Y, Z and P are matrices of order $2\times n, 3\times k, 2\times p, p\times 3$ and $p\times k$, respectively. Find the restriction on n, k and p so that PY + WY will be defined.

SECTION – B

Q11. If $f(x)=\begin{cases}|x|\cos\frac{1}{x}, & if \quad x\neq 0\\ 0 \quad , & if \quad x=0\end{cases}$ then discuss the continuity of f(x) at x = 0.

OR

If $y.\sqrt{x^2+1} = log\left[\sqrt{x^2+1} - x\right]$ then show that $\left(x^2+1\right)\frac{dy}{dx} + xy + 1 = 0$

Q12. If $\vec{a}, \vec{b}, \& \vec{c}$ are three unit vectors such that $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c} = 0$ and the angle between $\vec{b} \& \vec{c}$ is $\frac{\pi}{6}$, prove that $\vec{a} = \pm 2\left(\vec{b}\times\vec{c}\right)$

Q13. Show that $\Delta = \begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$

Q14. If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, find $\frac{d^2y}{dx^2}$.

Q15. (i) Evaluate : $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\frac{x-y}{x+y}$

(ii) Solve the following equation : $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1}x, (x>0)$

Q16. Define a binary operation * on the set {0, 1, 2, 3, 4, 5} as

$$a * b = \begin{cases} a+b, & \text{if } a+b<6 \\ a+b+6 & \text{if } a+b \geq 6 \end{cases}$$

Show that zero is the identity for this operation and each element a of the set is invertible with 6 – a being the inverse of a.

OR

Let $f : X \to Y$ be an invertible function. Show that f has unique inverse.

Q17. Evaluate: $\int_{-\pi/2}^{\pi/2} \log\left|\frac{2-\sin x}{2+\sin x}\right| dx.$

OR

Evaluate: $\int_0^{\pi/2} (2\log\sin x - \log\sin 2x)dx.$

Q18. Evaluate : $\int \frac{1}{\cos(x-a)\cos(x-b)}dx$

Q19. Show that the four points (0, -1, -1), (4, 5, 1), (3, 9, 4) and (-4, 4, 4) are coplanar. Also, find the equation of plane containing them.

Q20 How many times must a man toss a fair coin, so that the probability of having at least one head is more than 80% ?

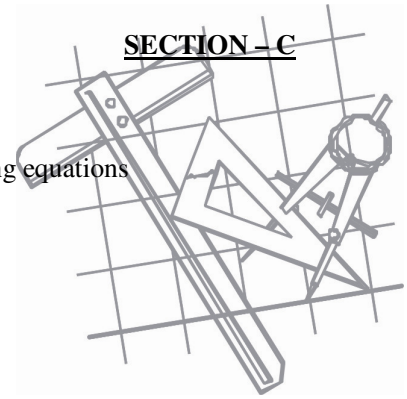
Q21. Evaluate : $\int_{-1}^{1} \left| x\cos(\pi x) \right| dx$

Q22. Find the equations of the tangent and normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_0, y_0).

OR

Two equal sides of an isosceles triangle with fixed base 'a' are decreasing at the rate of 9 cm /sec. How fast is the area of the triangle decreasing when two sides are equal to 'a'.

SECTION – C



Q23. Solve the system of the following equations

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

Q24. Draw a sketch of the following region and find its area : $\{(x, y) : x^2 + y^2 \le 1 \le x + y\}$

Q25. A firm manufactures two types of products A and B and sells them at a profit of Rs.5 per unit of type A and Rs.3 per unit of type B. Each product is produced on two machines M_1 and M_2. One unit of type A requires one minute of processing time on M_1 and two minutes of processing time on M_2, where as one unit of type B requires one minute of processing time on M_1 and one minute on M_2. Machines M_1 and M_2 are respectively available for at most 5 hours and 6 hours in a day. Find out how many units of each type of product should the firm produce a day in order to maximize the profit. Solve the problem graphically.

Q26. A fair die is rolled. If 1 turns up, a ball is picked up at random from bag A, if 2 or 3 turns up, a ball is picked up at random from bag B, otherwise a ball is picked up at random from bag C. Bag A contains 3 red & 2 white balls, Bag B contains 3 red & 4 white balls and Bag C balls contains 4 red & 5 white balls. The die is rolled, a bag is picked & a ball is drawn from it. If the ball drawn is red, what is the probability that bag B was picked up.

Q27. A window is in the form of a rectangle surmounted by semicircular opening. The total perimeter of the window is 10m. Find the dimensions of the window to admit maximum light through the whole opening.

OR

If lengths of three sides of a trapezium other than base are equal to 10 cm, then find the area of the trapezium when it is maximum.

Q28. Find the equation of the plane passing through the point (1, 1, 1) and containing the line $\vec{r}=\left(-3\hat{i}+\hat{j}+5\hat{k}\right)+\lambda\left(3\hat{i}-\hat{j}+5\hat{k}\right)$. Also show that the plane contains the line $\vec{r}=\left(-\hat{i}+2\hat{j}+5\hat{k}\right)+\lambda\left(\hat{i}-2\hat{j}-5\hat{k}\right)$

Q29. (i) Show that the differential equation, of which $x^2-y^2=c(x^2+y^2)^2$ is a solution is:

$$(x^3-3xy^2)dx=(y^3-3x^2y)dy$$

(ii) Find the particular solution of the differential equation

$$(xdy-ydx)\,y\cdot\sin\left(\frac{y}{x}\right)=(ydx+xdy)\,x\cos\left(\frac{y}{x}\right)$$, given that $y(3)=\pi$