SANGAM SCHOOL OF EXCELLENCE, BHILWARA

I PRE-BOARD EXAM– 2010

Class – XII | Max Time: 3Hrs

Subject – Math | Max Marks: 100

General Instructions:

1. All questions are compulsory
2. This question paper consists of **29** questions divided into three sections **A**, **B** and **C**. Section A comprises of **10** questions of **one mark** each, section **B** comprises of **12** questions of **four marks** each and section **C** comprises of **07** questions of **six marks** each
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question
4. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

SECTION – A

1. Find the value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.
2. Find the value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right]$.
3. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$
4. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, Find x and y.
5. If siny = xsin(a+y),then find dy/dx at (0,0) .
6. Find the value of $\int_0^{\pi/2} \sin^2 x\,dx$.
7. Check the continuity of the function $f(x) = \begin{cases} 1, & if \ x \le 0 \\ 2, & if \ x > 0 \end{cases}$
8. $\int (x + \cos 6x)/(3x^2 + \sin 6x)\,dx$
9. Find the maximum and minimum values of $|\sin x + 3|$
10. Find the approximate value of $\sqrt{25.3}$

SECTION – B

11. Prove that: $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy + yz + zx)$.

    Or

    If x,y,z are different and $A = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then show that 1+xyz=0.

12. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k, so that $A^2 = Ak - 2I$

    Or

    Without expanding find the value of the following determinant,

$$\Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix}$$

13. Find the value of 2A-3B+5C = 0 where,

$A=\begin{bmatrix}1 & 2 & 3\\ -5 & 6 & -7\\ 0 & 1 & 5\end{bmatrix}$, $B=\begin{bmatrix}-2 & 2 & 0\\ 3 & 1 & 4\\ 0 & -1 & 5\end{bmatrix}$ & $C=\begin{bmatrix}2 & 0 & -2\\ 7 & 1 & 6\\ 1 & 0 & 0\end{bmatrix}$

14. Using Principle of Mathematical Induction, prove that

$A^n=\begin{bmatrix}1+2n & -4n\\ n & 1-2n\end{bmatrix}$ Where, $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$

15. Evaluate $\int_{-1}^{3/2} x\sin\pi x\,dx$

Or

Evaluate $\int_0^{\pi/2}\log(\cos x)\,dx$

16. If $A=\begin{bmatrix}-2\\ 4\\ 5\end{bmatrix}$ and $B=\begin{bmatrix}1 & 3 & -6\end{bmatrix}$, verify that $(AB)'=B'A'$

17. If $\sin y = x\sin(a+y)$, show that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$

18. The volume of a spherical balloon is increasing at the rate of $20cm^3/sec$. Find the rate of change of its surface area at the instant when its radius is 8cm.

Or

Verify mean value theorem for f(x) =x(x-1)(x-2) in the interval $\left[0,\frac{1}{2}\right]$.

19. If x=a(α-sin α), y=a(1+cos α), find $\frac{d^2y}{dx^2}$ at $\alpha=\pi/2$

20. Solve for x : $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$

21. Show that: $\sin^{-1}(3/5)-\sin^{-1}(8/17)=\cos^{-1}(84/85)$

22. Find $\frac{dy}{dx}$ for $xy+y^2=\tan x+y$.

SECTION – C

23. Prove that the volume of the largest cone that can be inscribed in a sphere of radium R is 8/27 of the volume of the sphere.

Or

Evaluate (i) $\int\frac{dx}{\sqrt{\sin^3 x\sin(x+\theta)}}$ (ii) $\int e^x\left[\frac{2+\sin 2x}{1+\cos 2x}\right]dx$

24. Find dy/dx if $y^x+x^y+x^x=a^b$

25. Using matrices solve the system of equations: $x+y+z=6$, $y+3z=11$, $x-2y+z=0$.

26. If $y=\left(\sin^{-1}x\right)^2$ show that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=2$

Or If $y\sqrt{1-x^2}+x\sqrt{1-y^2}=1$, show that $\frac{dy}{dx}=-\sqrt{\frac{1-y^2}{1-x^2}}$

27. Express the following matrix B as the sum of symmetric and skew symmetric matrix:

$B=\begin{bmatrix}2 & -2 & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{bmatrix}$

28. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2m and volume is $8m^3$. If building of tank cost Rs. 70 per square meters for the base and Rs. 45 per square meters for sides. What is the cost of least expensive tank?

29. Prove that:

$$\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right]=\frac{x+y}{1-xy},\ |x|<1, y>0 xy<1.$$