Er Manish Bhadoria's
Interactions
Study Circle
Strong Foundation for a bright future

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Mathematics

(Answers and Solutions to Sample Paper - II)

This is the answer/solution sheet for the Mathematics sample paper – 02 prepared by me and published on www.cbseguess.com on 03 Feb 2010.

Perma link for the question paper is:
http://www.cbseguess.com/papers/paper_description.php?paper_id=2548

Answers

1. 250
2. $-\frac{4}{3}$
3. $-\frac{b}{a}$
4. D > 0
5. 50 cm
6. 5.5 cm
7. Median
8. $\frac{1}{6}$
9. 50°
10. 30°
11. – 78
12. ---
13. 18.33 cm
14. 12 sq. units
15. $\frac{11}{75}$
16. 2 and – 3
17. 60 days
18. a = - 1, b = 2
    Choice
    Fixed charge = Rs 10, Charge for each extra day = Rs 3/day
19. x = 40
    Choice
    no. of rows = 15, no. of plants in last row = 12
20. ---
21. $(-1, \frac{7}{2})$, (0, 5), $(1, \frac{13}{2})$
    Choice
    3, 5
22. $(-\frac{7}{5}, \frac{12}{5})$
23. ---
24. ---
25. 75.625 m^2
26. Larger pipe: 20 h, Smaller pipe: 30 h
27. ---
28. ---
29. 170.8 cm^3
    Choice
    562500 m^2
30. $f_1 = 28$, $f_2 = 24$

Hints / Solutions

1. HCF (x, y) × LCM(x, y) = x × y
   5 × 1750 = 35 × y $\Rightarrow$ $y = \frac{5 \times 1750}{35} = 250$ *Ans*

2. Let $p(x) = (k – 1)x^2 + kx + 1$
   ∵ - 3 is a zero of p(x)
   ∴ p(-3) = 0
   Or, $(k – 1)(-3)^2 + k(–3) + 1 = 0$
   Solving, $k = -\frac{4}{3}$ *Ans*

3. For a cubic polynomial,
   $\alpha + \beta + \gamma = -\frac{b}{a}$ $\Rightarrow$ $0 + 0 + \gamma = -\frac{b}{a}$ $\Rightarrow$ $\gamma = -\frac{b}{a}$ *Ans*

4. D > 0

5. Let radius of big circle be r. Then, according to the problem –
   $\pi r^2 = \pi (24)^2 + \pi (7)^2$ $\Rightarrow$ r = 25 $\Rightarrow$ Diameter = 50 cm *Ans*

6. We'll make use of the fact that tangents drawn from an external point to a circle are equal in length. . So, PA = PB = 8 cm and AC = CQ = 2.5 cm.
   AP = AC + CP = 8 $\Rightarrow$ CQ + CP = 8 $\Rightarrow$ 2.5 + CP = 8 $\Rightarrow$ CP = 5.5 cm *Ans*

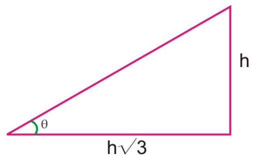
7. Median

8. Favourable events are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) [total 6 no.]
   All events are 36 in number.
   ∴ P(E: same no. on both dies) = $\frac{6}{36} = \frac{1}{6}$ *Ans*

9. ∠ABC = 90° (angle in a semicircle)
   In ΔABC, ∠ACB + ∠ABC + ∠CAB = 180° (angle sum property of a triangle)
   50° + 90° + ∠CAB = 180° $\Rightarrow$ ∠CAB = 40°.
   ∠CAT = 90° (angle between radius and tangent at the point of contact of tangent is a right angle)
   ∠CAT = ∠CAB + ∠BAT = 90° $\Rightarrow$ 40° + ∠BAT = 90° $\Rightarrow$ ∠BAT = 50° *Ans*

10. $\tan\theta = \frac{h}{h\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30°$

$\therefore\ \theta = 30°$ *Ans*



11. Writing the AP in reverse order: -100, -98, -96, ……

For this, we have to find 12th term from beginning.

a = -100, d = 2

$a_{12} = a + 11d = -100 + 11(2) = -100 + 22 = -78$ *Ans*

12. $\sin\theta + \cos\theta = \sqrt{3} \Rightarrow (\sin\theta + \cos\theta)^2 = 3 \Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

Or, $1 + 2\sin\theta\cos\theta = 3 \Rightarrow 2\sin\theta\cos\theta = 2 \Rightarrow \sin\theta\cos\theta = 1$ ……(i)

$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} = \frac{1}{1} = 1$ *Proved*

Or

$\frac{\cos^2(45^\circ+\theta)+\cos^2(45^\circ-\theta)}{\tan(60^\circ+\theta)\tan(30^\circ-\theta)} = \frac{\sin^2[90^\circ-(45^\circ+\theta)]+\cos^2(45^\circ-\theta)}{\cot[90^\circ-(60^\circ+\theta)]\tan(30^\circ-\theta)}$

$= \frac{\sin^2(45^\circ-\theta)+\cos^2(45^\circ-\theta)}{\cot(30^\circ-\theta)\tan(30^\circ-\theta)} = \frac{1}{\frac{1}{\tan(30^\circ-\theta)}\times\tan(30^\circ-\theta)} = 1$ *Proved*

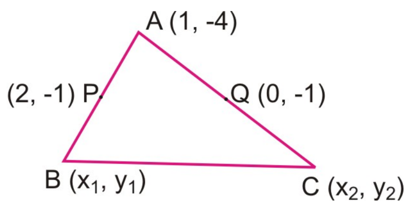
13. In ΔABC and ΔACD, ∠ACB = ∠CDA (given) and ∠A = ∠A (common).

∴ΔACB ~ ΔADC (AA similarity)

So, $\frac{AC}{AD} = \frac{AB}{AC} \Rightarrow AB = \frac{AC^2}{AD} = \frac{64}{3}$ cm

Now, $BD = AB - AD = \frac{64}{3} - 3 = \frac{55}{3}$ cm *Ans*

14. Let coordinates of B and C be (x_1, y_1) and (x_2, y_2) respectively.



Then, $\frac{1+x_1}{2} = 2$ and $\frac{-4+y_1}{2} = -1$

Solving, $x_1 = 3$, $y_1 = 2$

Similarly, $x_2 = -1$ and $y_2 = 2$

$ar(\Delta ABC) = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[1(2 - 2) + 3(2 + 4) - 1(-4 - 2)] = 12$ sq. units *Ans*

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15. After pouring the contents of both the boxes into a third box-

Total no. slips = 75, No. of slips of Re 1 = 19 + 45 = 64, No. of slips of Rs 5 = 6 and No. of slips of Rs 13 = 5.

P(E: Re 1 slip) = $\frac{64}{75} \Rightarrow$ P(E: not Re 1) = $1 - \frac{64}{75} = \frac{11}{75}$ *Ans*

16. [note – steps discussed in brief only]

Since √3 and -√3 are zeroes of p(x), (x - √3) and (x + √3) are factors of p(x).

∴ (x - √3)(x + √3) is a factor of p(x) ⇒ $(x^2 - 3)$ is a factor of p(x).

Dividing p(x) by $x^2 - 3$ by long division method, we get remainder equal to zero and quotient equal to $x^2 + x - 6$. This quotient is the other factor of p(x).

Then $p(x) = (x^2 - 3)(x^2 + x - 6)$.

$x^2 + x - 6 = (x + 3)(x - 2)$

∴ Other factors of p(x) are -3 and 2. *Ans*

17. Time = $\frac{\text{Distance}}{\text{Speed}}$

So time taken by different cyclist in completing one round-

For first cyclist, $t_1 = \frac{360}{60} = 6$ days

For second cyclist, $t_2 = \frac{360}{72} = 5$ days

For third cyclist, $t_3 = \frac{360}{90} = 4$ days.

Finding LCM(6, 5, 4):

6 = 2 × 3

5 = 5

$4 = 2^2$

∴ LCM(6, 5, 4) = $2^2 \times 3 \times 5 = 60$ days *Ans*

18. Condition for infinite solutions:

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{4}{2a+7b} = \frac{5}{a+8b} = \frac{2}{2b-a+1}$

Two equations are obtained from these.

2a + b = 0……(i)

7a + 6 b = 5……(ii)

Solving these equations, a = - 1, b = 2 *Ans*

Or

Let the fixed charge be Rs x (for first two days) and additional charge be Rs y (for each extra day).

Total charge for 6 days: x + 4y = 22……(i)

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Total charge for 4 days = x + 2y = 16……(ii)
Solving these equation, x = 10 and y = 3.
So, fixed charge = Rs 10 and additional charge = Rs 3 per day. *Ans*

19. The terms of this series are in AP. [a = 1, d = 3]
Let there be total n terms in the series. Then, x = n^{th} term (a_n)
x = a + (n – 1)d = 1 + (n – 1)3 = 3n – 2
Given that S_n = 287

$\frac{n}{2}$[2a + (n – 1)d] = 287 ⇒ $\frac{n}{2}$[2 + (n – 1)3] = 287

Or, $\frac{n}{2}$[3n – 1] = 287 ⇒ $3n^2 – n – 574 = 0$

Solving this quadratic equation, n = 14 and n = - $\frac{41}{3}$.
Number of terms in an AP cannot be negative (and fractional number).
∴ n = 14.
Then, x = 3n – 2 = 3(14) – 2 = 42 – 2 = 40 *Ans*

Or

40 + 38 + 36 + …… = 390
It's an AP with a = 40 and d = -2.
Let number of rows = n

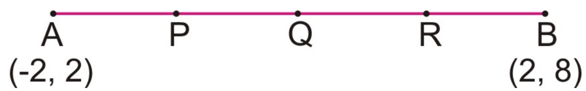
Then, S_n = 390 ⇒ $\frac{n}{2}$[2a + (n – 1)d] = 390 ⇒ $\frac{n}{2}$[80 + (n – 1)(-2)] = 390

Or, $\frac{n}{2}$[-2n + 82] = 390 ⇒ n(-n + 41) = 390 ⇒ $-n^2 + 41n – 390 = 0$
Solving, n = 15 or n = 26
If we take n = 26, then number of plants in this row,
a_{26} = a + 25d = 40 + 25(-2) = -10, which is impossible.
So, number of rows = 15 *Ans*
Then number of plants in last (i.e. 15^{th})row = a + 14d = 40 + 14(-2) = 12 *Ans*

20. LHS = $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta} = \frac{(\sec^2\theta-\tan^2\theta)+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}$

$= \frac{(\sec\theta-\tan\theta)(1+\sec\theta+\tan\theta)}{1+\sec\theta+\tan\theta}$ = sec θ - tan θ = $\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = \frac{1-\sin\theta}{\cos\theta}$ =
RHS *Proved*

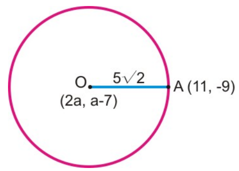
21. Let the points which divide line segment AB in four equal parts be P, Q and R.



Point P divides AB in the ratio 1 : 3
Using section formula, coordinates of P are
x = $\frac{1\times2+3(-2)}{1+3}$ = -1, y = $\frac{1\times8+3\times2}{1+3} = \frac{7}{2}$
Point Q is the mid-point of AB. Therefore coordinates of Q are-
x = $\frac{-2+2}{2}$ = 0, y = $\frac{2+8}{2}$ = 5
Point R divides AB in the ratio 3 : 1
∴ Coordinates of R are
x = $\frac{3\times2+1\times(-2)}{4}$ = 1, y = $\frac{3\times8+1\times2}{4} = \frac{13}{2}$
So, P(-1, $\frac{7}{2}$), Q(0, 5), R(1, $\frac{13}{2}$) *Ans*

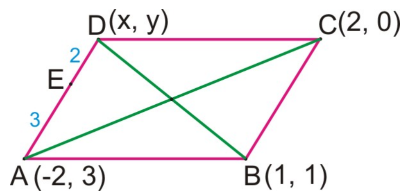
Or

Let A be the point (11, -9) on the circumference of the circle.



Given, diameter = 10√2 units
Then, OA = radius = 5√2 units
By distance formula:
OA = $\sqrt{(2a-11)^2+(a+2)^2}$ = 5√2
Solving, a = 5 or a = 3 *Ans*

22. Let coordinates of point D be (x, y).
In a parallelogram, diagonals bisect each other.
∴ mid-point of AC is same as mid-point of BD.



$\frac{x+1}{2} = \frac{2-2}{2}$ ⇒ x = -1 and $\frac{y+1}{2} = \frac{3+0}{2}$ ⇒ y = 2

Now, given that AE = $\frac{3}{5}$ AD ⇒ AE = $\frac{3}{5}$(AE + ED) ⇒ AE - $\frac{3}{5}$AE = $\frac{3}{5}$ED

⇒ $\frac{2}{5}$AE = $\frac{3}{5}$ED ⇒ $\frac{AE}{ED} = \frac{3}{2}$.

Then coordinates of E (by section formula) are:

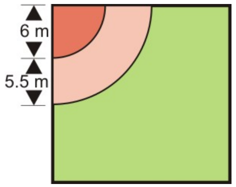
Abscissa = $\frac{2(-2)+3(-1)}{3+2} = -\frac{7}{5}$ and Ordinate = $\frac{2\times3+3\times2}{5} = \frac{12}{5}$

So, point E is $(-\frac{7}{5}, \frac{12}{5})$ *Ans*

23. ---

24. Lengths of tangents drawn from an external point to a circle are equal.
AQ = AB + BQ = AB + BP……(i)
AR = AC + CR = AC + CP
But AQ = AR
∴ AQ = AC + CP……(ii)
Adding (i) and (ii) –
2 AQ = AB + BP + PC + AC = AB + BC + AC
∴ AQ = ½(AB + BC + AC) *Proved*

25. Increase in graze area = area of bigger quadrant of circle – area of smaller quadrant



Increase = $\frac{1}{4}\pi(11.5)^2 - \frac{1}{4}\pi(6)^2$

$= \frac{1}{4} \times \frac{22}{7}(17.5 \times 5.5) = 75.625\ m^2$ *Ans*

26. Let the volume of the pool be V.
Also let the larger pipe fills the pool in x hours and the smaller pipe fills it in y hours.

By larger Pipe:

In x hours volume filled = V

∴ In 1 hour volume filled = $\frac{V}{x}$

∴ In 12 hours volume filled = $\frac{12V}{x}$

By Smaller Pipe:

In 12 hours volume filled = $\frac{12V}{y}$

When both pipes run together volume filled in 12 hours = V

i.e., $\frac{12V}{x} + \frac{12V}{y} = V$

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Or, $\frac{1}{x} + \frac{1}{y} = \frac{1}{12}$ ……(i)

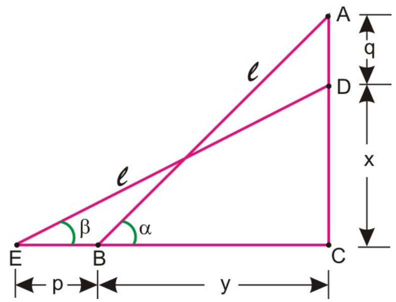
According to second condition-

$\frac{4V}{x} + \frac{9V}{y} = \frac{V}{2}$

Or, $\frac{4}{x} + \frac{9}{y} = \frac{1}{2}$ ……(ii)

Solving equation (i) and (ii) –
x = 20 and y = 30
Thus, larger diameter pipe takes 20 hours and smaller diameter pipe takes 30 hours to fill the pool. *Ans*

27. Let AB be the ladder of length l resting against wall AC. Given, ∠ABC = α. When its foot is pulled away through a distance p, its top end slides down a distance q. Now the ladder takes the position DE and ∠DEC = β.
Also let DC = x and BC = y.



In ΔABC, $\cos\alpha = \frac{y}{l}$ and in ΔDEC, $\cos\beta = \frac{p+y}{l} = \frac{p}{l} + \frac{y}{l} = \frac{p}{l} + \cos\alpha$

So, $\cos\beta - \cos\alpha = \frac{p}{l} \Rightarrow p = l(\cos\beta - \cos\alpha)$……(i)

In ΔDEC, $\sin\beta = \frac{x}{l}$ and in ΔABC, $\sin\alpha = \frac{q+x}{l} = \frac{q}{l} + \frac{x}{l} = \frac{q}{l} + \sin\beta$

So, $\sin\alpha - \sin\beta = \frac{q}{l} \Rightarrow q = l(\sin\alpha - \sin\beta)$……(ii)

Dividing (i) by (ii), $\frac{p}{q} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$ *Proved*

28. BPT.

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EF || AB || DC (given)

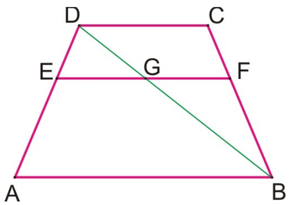
In ΔDAB, EG || AB

$\therefore \frac{AE}{ED} = \frac{BG}{GD}$ (By BPT)……(i)

In ΔBDC, GF || DC

$\therefore \frac{BG}{GD} = \frac{BF}{FC}$ (By BPT)……(ii)

From (i) and (ii), $\frac{AE}{ED} = \frac{BF}{FC}$ *Proved*



Or

Pythagoras theorem.

A rhombus is a parallelogram in which all sides are equal and diagonals bisect each other at right angle.

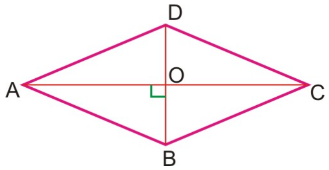
$AB^2 = OA^2 + OB^2$ (By Pythagoras theorem)

$= (½ AC)^2 + (½ BD)^2$

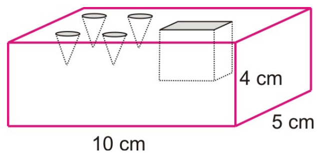
$= \frac{AC^2}{4} + \frac{BD^2}{4}$

Or, $4\ AB^2 = AC^2 + BD^2$

Proved



29. Volume of wood in the pen stand
= volume of cuboid – volume of four conical depressions – volume of one cubical depression



$= 10 \times 5 \times 4 - 4 \times \frac{1}{3} \times \frac{22}{7} \times (0.5)^2(2.1) - (3)^3$

$= 200 - 2.2 - 27 = 170.8\ cm^3$ *Ans*

Or

Dimensions of canal: width = 6 m, height = 1.5 m

Speed of water in the canal = 10 km/h

Distance covered by water in the canal in 30 min (= ½ hour)

Distance = speed × time

= 10 km/h × ½ h = 5 km = 5000 m

∴ Volume of water passed through the canal in 30 min

= length × breadth × height

$= 5000 \times 6 \times 1.5 = 45{,}000\ m^3$

Height of water required in field = 8 cm = 0.08 m

Volume of water collected in field = volume of water passed through canal

∴ Area irrigated × height = 45,000

Area = $\frac{45000}{0.08}$ = 562500 m^2 *Ans*

30.

Class	Class Mark (x_i)	Frequency (f_i)	f_ix_i
0-20	10	17	170
20-40	30	f_1	30 f_1
40-60	50	32	1600
60-80	70	f_2	70 f_2
80-100	90	19	1710
Total		$\sum f_i = 68 + f_1 + f_2$	$\sum f_ix_i = 3480 + 30f_1 + 70f_2$

Given, $\sum f_i = 120$

$\therefore 68 + f_1 + f_2 = 120 \Rightarrow f_1 + f_2 = 52$……(i)

Also given, mean = 50

$\overline{X} = \frac{\sum f_ix_i}{\sum f_i}$

$50 = \frac{3480 + 30f_1 + 70f_2}{120}$

On simplifying, $3f_1 + 7f_2 = 252$……(ii)

Solving eqn. (i) and (ii), $f_1 = 28$ and $f_2 = 24$ *Ans*

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