Year of Quality education 18th

Code No. **Series AG-F6** **TMG -D/79/89**

- Please check that this question paper contains 4 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 29 questions.

General Instructions: -

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A contains 10 questions of 1 marks each, Section B is of 12 questions of 4 marks each and Section C is of 7 questions of 6 marks each.
3. Write the serial number of the question before attempting it.
4. If you wish to answer any question already answered, cancel the previous answer.
5. In questions where internal choices is provided. You must attempt only one choice.

Pre-Board Examination 2009 -10

Time: 3 hrs. ***M.M.: 100***

CLASS – XII *MATHEMATICS*

Section A

Q.1	If the graph of $y = f(x)$ is given and the line parallel to x-axis cuts the curve at more than one point .Write the name of function .
Q.2	The vectors $\vec{a} = 3\hat{i} + x\hat{j} - \hat{k}$ & $\vec{b} = 2\hat{i} + \hat{j} + y\hat{k}$ are mutually perpendicular. Given that $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert$, find the values of x and y .
Q.3	If A is a matrix of order $m \times n$ and C is a column of A, find order of R as a matrix.
Q.4	Prove that: $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$.
Q.5	If A = $\begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then find the value of x .
Q.6	If f(x) is real function, then find $\int_0^{2a} \frac{f(x)}{f(x) + f(2a - x)} dx$.
Q.7	Find c if Rolle's Theorem are verified by f(x)=$e^x(\sin x - \cos x)$ on $\left[\frac{\pi}{4} \quad \frac{5\pi}{4}\right]$.
Q.8	Let $f : R \to R$ be defined by $f(x) = x^2 - 3x + 1$, Find $f[f(x)]$.
Q.9	Find x, y if the points (x, -1, 3), (3, y, 1) and (-1, 11, 9) are collinear.
Q.10	Find the condition that the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ may be invertible .

Resi.: D-79 Vasant Vihar ; Office : 89-Laxmi bai colony
Ph. :2337615; 4010685®, 92022217922630601(O) Mobile : 9425109601;9907757815 (P); 9300618521;9425110860(O);9993461523;9425772164

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Section B

Q.11 Let $f: N \rightarrow R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f: N \rightarrow S$, where, S is the range of f, is invertible. Find the inverse of f.

Q.12 Find the equation of the tangent to the curve y = $\sqrt{3x-2}$ which is parallel to the line 4x – 2y +5 = 0 .

Q.13 Using the properties of determinants, prove that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$.

Q.14 Evaluate : $\int_{-5}^{0} f(x)\,dx$, where $f(x) = |x| + |x+3| + |x+6|$.

Q.15 Prove that f(x) = $\frac{e^{1/x} - 1}{e^{1/x} + 1}$, does not exist at x = o .

Q.16 Solve the differential equation $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + x\,dy = 0;\ y = \frac{\pi}{4}\ when x = 1$.

Q.17 (I) Evaluate : $\int \frac{dx}{x(x^8+1)}$.(II) Evaluate : $\int \sec^3 x\ dx$.

Q.18 In a school, there are 1000 students, out of which 430 are girls. It is known that out of 430, 10% of girls study in class XII. What is the probability that a student chosen randomly studies in class XII given that the chosen student is a girl ?

Or

Two balls are drawn at random from a bag containing 3 white, 3 red, 4 green and 4 black balls one by one without replacement. Find the probability that both the balls are different colors.

Q.19 Using vectors, prove that $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$.

Or

Using vectors prove that the altitudes of a triangle are concurrent.

Q.20 If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, prove that $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$.

or

If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$

Q.21 Given that $\cos\left(\frac{x}{2}\right)\cdot\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{8}\right).... = \frac{\sin x}{x}$.

Show that: $\frac{1}{2^2}\cdot\sec^2\left(\frac{x}{2}\right) + \frac{1}{2^4}\cdot\sec^2\left(\frac{x}{4}\right) + \frac{1}{2^8}\cdot\sec^2\left(\frac{x}{8}\right) + ... = \operatorname{cosec}^2 x - \frac{1}{x^2}$.

Or

If $x = \sin t$ and $y = \sin pt$, then prove that $(1-x^2)y_2 - xy_1 + p^2 y = 0$.

Q.22 Find the equations of the bisector planes of the angles between the planes $3x - 2y + 6z + 8 = 0$ and $2x - y + 2z + 3 = 0$. Also point out the bisector of the obtuse angles between the given planes.

Or

Find the vector equation of the plane $\vec{r} = (1+s-t)\hat{i} + (2-s)\hat{j} + (3-2s+2t)\hat{k}$ in the scalar product form.

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Section C

Q.23 Find the product of matrices $\begin{pmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{pmatrix}$ and $\begin{pmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{pmatrix}$. and use it for solving equations : $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4;\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1;\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2.$

Q.24 A manufacturer makes two type of toys A and B. Three machines are needed for this purpose and the time (in minutes) required for each toy on the machine is given below.

Type of Toys	Machines		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each toy of type A is Rs 7.50 and that on each toy of type B is Rs 5, show that 15 toys of type A and 30 toys of type B should be manufactured in a day to get maximum profit.

Q.25 Evaluate: $\int_0^{\infty} \sqrt{\tan x}dx.$

Q.26 Find the area of the region; $\{(x, y); x^2+ y^2 \le 1 \le x+y\}$.

Or

Draw the rough sketch of the region enclosed between the circles $x^2 + y^2 = 9$ and $(x-3)^2 + y^2 = 1$. Using integration, find the area of the enclosed region.

Q.27 Two bag A and B contains 4 white and 3 black balls and 2 white and 2 black balls respectively. From bag A, two balls are drawn at random and then transferred to bag B. A ball is then drawn from bag B and is found to be a black ball. What is the probability that the transferred balls were 1 white and 1 black?

Or

Assume that the chances of a patient having a heart attack are 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga ?

Q.28 Prove that the line of section of the planes $2x - y + z = 4 and 5x + 7y + 2z = 0$ meets the plane $3x + 4y - 2z + 3 = 0$ in a single point. Find the co-ordinates of that point .

Q.29 The sum of the surface areas of a rectangular parallelepiped with sides x , 2x and x / 3 and a sphere of radius r is given to be constant . Prove that sum of their volume is minimum if x = 3r .Also find the minimum value of the sum of the volumes .

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