Mathematics

Class X

TOPPER SAMPLE PAPER-1

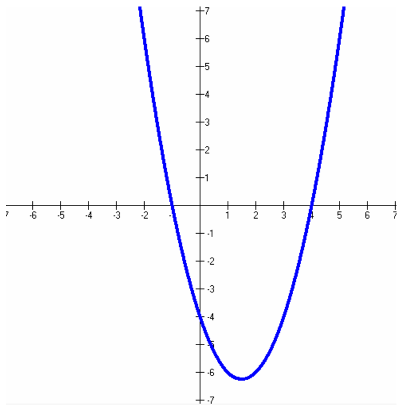
Maximum Marks: 80 **Time: 3 Hrs**

1. All questions are compulsory.
2. The question paper consist of 30questions divided into four sections A, B,C and D. Section A comprises of 10 questions of one mark each, section B comprises of 5 questions of two marks each ,section C comprises of 10 questions of three marks each and section D comprises of 5 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. In question on construction, the drawing should be neat and exactly as per the given measurements.
5. Use of calculators is not permitted. You may ask for mathematical tables, if required.
6. There is no overall choice. However, internal choice has been provided in one question of 02 marks each, three questions of 03 marks each and two questions of 06 marks each. You have to attempt only one of the alternatives in all such questions.

Section A

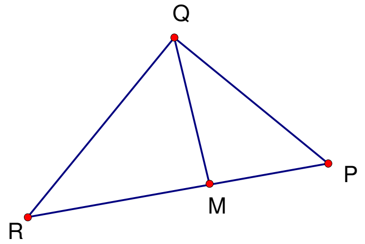
Q1 If HCF (252, 378) = 126, find their LCM.

Q2 Find the polynomial shown in the graph.

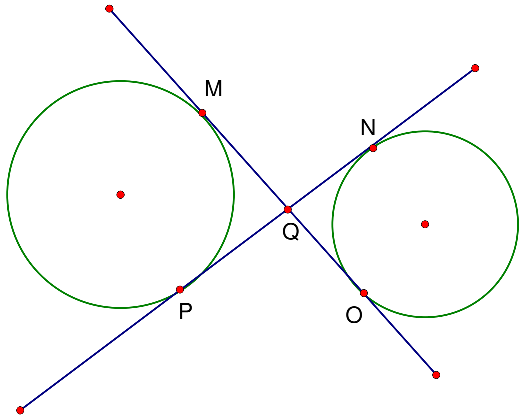


Q3 For what value of 'k' will the equations 13x+23y-1=0 and kx–46y-2=0 represents intersecting lines?

Q4 QM ⊥ RP and $PR^2 - PQ^2 = QR^2$. If ∠QPM = 30^0, find ∠MQR.



Q5 Find the length of PN if OM = 9 cm.



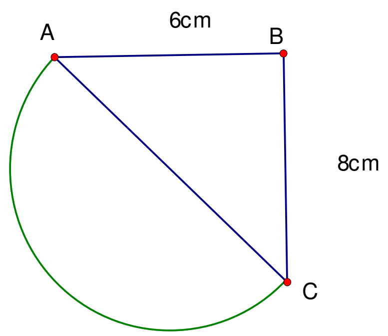
Q6 If the median of a data which represents the weight of 150 students in a school is 45.5 kg, find the point of intersection of the less than and more than ogive curves.

Q7 If two coins are tossed simultaneously, find the probability of getting exactly two heads.

Q8 If three times the third term of an AP is four times the fourth term , find the seventh term.

Q9 If $\sin\alpha + \cos\alpha = \sqrt{2}\cos(90 - \alpha)$, find $\cot\alpha$.

Q10 Find the perimeter of the figure , where AC is the diameter of the semi circle and AB $\perp$ BC



SECTION B

Q11 A and B are points (1, 2) and (4, 5) . Find the coordinates of a point P on AB if AP = $\frac{2}{5}$ AB.

Q12 ΔABC is right angled at C. Let BC = a , AB = c and AC = b. p is the length of the perpendicular from C to AB. Prove that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$

Q13 Solve for x and y : 3(2x+y) = 7xy, 3(x +3y) = 11xy

Q14 Find the probability of getting 5 Wednesdays in the month of August.

Q15 If sin (A + B) = 1 and cos (A - B)= $\frac{\sqrt{3}}{2}$, $0\le A+B\le 90°$, A >B , find A and B.

OR

If tan A = $\frac{7}{24}$, evaluate $\sqrt{\frac{1-\cos A}{1+\cos A}}$

SECTION C

Q16 Prove that $\sqrt{5}$ is an irrational number.

Q17 Find the coordinates of a point(s) whose distance from (0,5) is 5 units and from (0,1) is 3 units.

Q18 Prove $(\sin A+\cos ecA)^2+(\cos A+\sec A)^2=\tan^2 A+\cot^2 A+7$

OR

Prove $(1+\cot\theta-\cos ec\theta)(1+\tan\theta+\sec\theta)=2$

Q19 Solve for x and y : $\frac{10}{x+y}+\frac{4}{y-x}=-2$, $\frac{15}{x+y}-\frac{7}{y-x}=10, \quad x+y\neq 0, x\neq y$

OR

For what values of 'm' will $2mx^2-2(1+2m)x+(3+2m)=0$ have real and distinct roots?

Q20 Find the area of a triangle whose sides have (10, 5), (8, 5) and (6, 6) as the midpoints.

Q21 If α, β are the zeroes of the polynomial $3x^2 - 11x + 14$, find the value of $\alpha^2 + \beta^2$.

Q22 Prove that a parallelogram circumscribing a circle is a rhombus.

OR

Prove that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Q23 Draw a circle of radius 3.5 cm. Construct two tangents to the circle which are inclined to each other at $120°$.

Q24 A grassy plot is in the form of a triangle with sides 45m, 32m and 35m. One horse is tied at each vertex of the plot with a rope of length 14m. Find the area grazed by the three horses.

Q25 The 46th term of an AP is 25. Find the sum of first 91 terms.

SECTION D

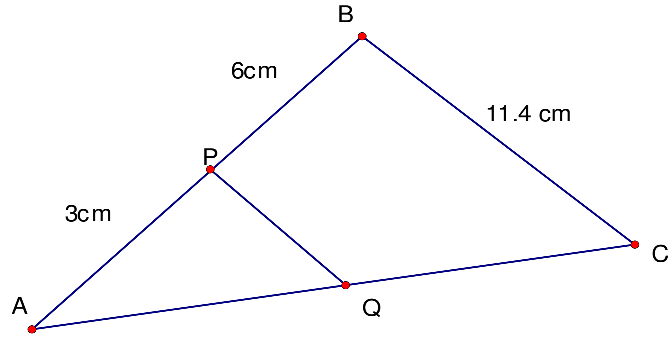
Q26 Prove that ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Using this theorem find the ratio of the area of the triangle drawn on the diagonal of a square and the triangle drawn on one side of the square.

OR

State and prove the Basic Proportionality Theorem. If PQ II BC, find

PQ.



Q27 The area of a rectangle remains the same if the length is increased by 7m and the breadth is decreased by 3 m. The area of the rectangle remains the same if the length is decreased by 7m and the breadth is increased by 5 m. Find the dimensions of the rectangle and the area of the rectangle.

Q28 A boy is standing on the ground and flying a kite with a string of 150m at an angle of $30°$. Another boy is standing on the roof of a 25m high building and flying a kite at an angle of $45°$. Both boys are on the opposite sides of the kites. Find the length of the string the second boy must have so that the two kites meet.

OR

At a point on level ground, the angle of elevation of a vertical tower is such that its tangent is $\frac{5}{12}$. On walking 192 m towards the tower, the tangent of the angle of elevation is $\frac{3}{4}$. Find the height of the tower.

Q29 A solid consists of a cylinder with a cone on one end and a hemisphere on the other end. If the length of the entire solid is 12.8cm and the diameter and height of the cylinder are 7cm and 6.5 cm respectively, find the total surface area of the solid.

Q30 Draw a less than ogive of the following data and find the median from the graph. Verify the result by using the formula.

Marks	Less than 140	Less than 145	Less than 150	Less than 155	Less than 160	Less than 165
No. of girls	4	11	29	40	46	51

Mathematics

Class X

TOPPER SAMPLE PAPER-1

SOLUTIONS

Ans1 HCF x LCM = Product of the 2 numbers

126 x LCM = 252 x 378

LCM = 756 (1 Mark)

Ans2 The zeroes are −1, 4

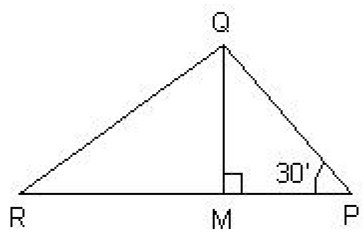
$\therefore \quad p(x) = (x+1)(x-4) = x^2 - 3x - 4$ (1 Mark)

Ans3 For intersecting lines:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{13}{k} \neq \frac{23}{-46}$ (1 Mark)

$\Rightarrow k \neq -26$

Ans4



Since $PR^2 - PQ^2 = QR^2$

$\Rightarrow PR^2 = QR^2 + PQ^2$

$\Rightarrow \angle RQP = 90^\circ$ (Converse of Pythagoras Theorem)

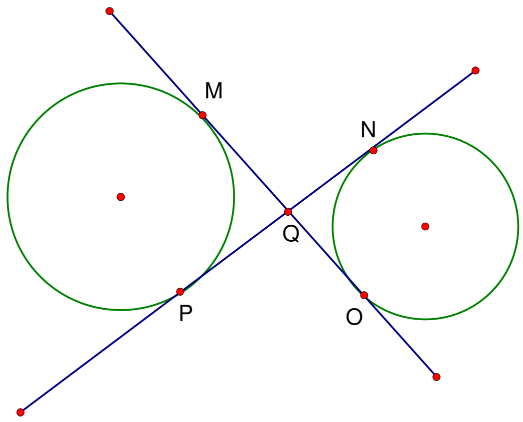
Therefore, In ΔPQM

Since $\angle QPM = 30^\circ$ and $\angle QMP = 90^\circ$

So $\angle MQP = 60^\circ$

Hence, $\angle MQR = 30^\circ$ (1 Mark)

Ans5



OM = MQ + QO
= QP + QN [Since Tangents from external point are equal]
= PN = 9cm (1 Mark)

Ans6 The two curves namely less than and more than ogives intersect at the median so the point of intersection is (45.5, 75) (1 Mark)

Ans7 Total outcomes = HH, TT, HT, TH

Favourable outcomes = HH

$P(E: Both\ Heads) = \frac{1}{4}$ (1 Mark)

Ans8 Let a_3 and a_4 be the third and fourth term of the AP
According to given Condition

$3.a_3 = 4.a_4$

$\Rightarrow 3(a+2d) = 4(a+3d)$

$\Rightarrow a = -6d$ (1 Mark)

$\Rightarrow a+6d = 0$

$\Rightarrow a_7 = 0$

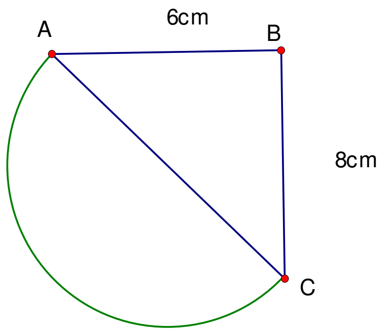
Ans9 $\sin\alpha + \cos\alpha = \sqrt{2}\,\sin\alpha$

$\cos\alpha = \sin\alpha\left(\sqrt{2}-1\right)$

$\dfrac{\cos\alpha}{\sin\alpha} = \sqrt{2}-1$ (1 Mark)

$\cot\alpha = \sqrt{2}-1$

Ans10



$AC = \sqrt{AB^2 + BC^2}$ (Using Pythagoras Theorem)

$= \sqrt{8^2 + 6^2}$

$= \sqrt{64+36}$

$= 10\text{ cm}$

Circumference of semi circle $= \pi r$

$= 3.14 \times 5$

$= 15.70 cm$

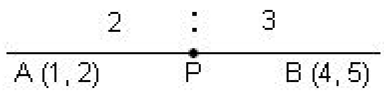
$\therefore$ Perimeter = 6 + 8 + 15 .7 (1 Mark)

= 29.7 cm

SECTION B

Ans11 Since, AP = $\frac{2}{5}$ AB

So AP: PB =2: 3

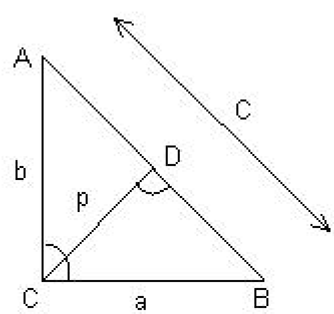


P divides AB in 2:3 ratios (1 mark)

$$P\left(\frac{2\times4+3\times1}{5},\ \frac{2\times5+3\times2}{5}\right)$$ $\left(\frac{1}{2}\text{mark}\right)$

$$P\left(\frac{11}{5},\frac{16}{5}\right)$$ $\left(\frac{1}{2}\text{mark}\right)$

Ans12



Area $(\Delta ACB)=\frac{1}{2}\,AC\,.\,CB$

$$=\frac{1}{2}\ a.b$$

Also, area $\left(\Delta ACB\right)=\frac{1}{2}.\ AB\ .CD$ $\left(\frac{1}{2}\text{mark}\right)$

$$=\frac{1}{2}\text{ cp}$$

$$\Rightarrow \quad \frac{1}{2}ab = \frac{1}{2}cp$$

$$\Rightarrow \quad ab = cp$$

$\left(\frac{1}{2}\text{mark}\right)$

Now $\frac{1}{a^2} + \frac{1}{b^2} = \frac{b^2 + a^2}{a^2b^2}$

$= \frac{c^2}{a^2b^2}$ (By Pythagoras theorem)

$= \frac{c^2}{a^2b^2}$

$= \frac{c^2}{c^2p^2}$ (Since, $ab = cp$)

$= \frac{1}{p^2}$

Hence Proved (1 Mark)

Ans13 $3(2x+y) = 7xy \Rightarrow 6x+3y = 7xy$ (1)

$3(x+3y) = 11xy \Rightarrow 3x+9y = 11xy$ (2)

Eq (2) $\times$ 2 gives: $6x+18y = 22xy$ (3)

When $x \neq 0$ and $y \neq 0$ eq(1) $-$ eq(3) gives

$-15y = -15xy$ $\left(\frac{1}{2}\text{mark}\right)$

$\Rightarrow \quad x = 1$ $\left(\frac{1}{2}\text{mark}\right)$

$\Rightarrow \quad y = \frac{3}{2}$ $\left(\frac{1}{2}\text{mark}\right)$

Also $x = 0, y = 0$ is a solution. $\left(\frac{1}{2}\text{mark}\right)$

Ans14 August has 31 days

$\Rightarrow$ 4 weeks and 3 days.

So 4 weeks means 4 Wednesdays

Now remaining 3 days can be

S M T	T W Th	Th F Sa	Sa. S M	(1 Mark)
M T W	W Th F	F Sa S		

Favorable outcomes are = M T W

T W Th $\left(\frac{1}{2}\text{mark}\right)$

W Th F

$\therefore$ P (3 Wednesdays) $=\frac{3}{7}$ $\left(\frac{1}{2}\text{mark}\right)$

Ans15 sin (A + B) = 1

Since $\sin 90^{\circ}=1$

A + B = 90° (1) $\left(\frac{1}{2}\text{mark}\right)$

$\cos(A-B)=\frac{\sqrt{3}}{2}$

since $\cos 30^{o} = \frac{\sqrt{3}}{2}$

A – B = 30° (2) $\left(\frac{1}{2}\text{mark}\right)$

Solving (1) and (2)

A = 60° $\left(\frac{1}{2}\text{mark}\right)$

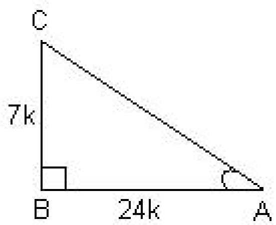
B = 30° $\left(\frac{1}{2}\text{mark}\right)$

OR

$\tan A = \dfrac{7}{24}$

So the ratio of adjacent and opposite side of the triangle is in the ratio 7:24

Let the common ratio term be k



Using Pythagoras Theorem

AC = 25 k. $\left(\dfrac{1}{2}\text{mark}\right)$

Consider $\sqrt{\dfrac{1-\cos A}{1+\cos A}}$

$$= \sqrt{\dfrac{1-\dfrac{24}{25}}{1+\dfrac{24}{25}}}$$ (1 Mark)

$$= \sqrt{\dfrac{1}{49}} = \dfrac{1}{7}$$ $\left(\dfrac{1}{2}\text{mark}\right)$

SECTION C

Ans16 Let us assume $\sqrt{5}$ is rational.

$\Rightarrow \sqrt{5} = \dfrac{p}{q}$ Where p and q are co prime integers and $q \neq 0$

$\left(\dfrac{1}{2}\text{mark}\right)$

$\Rightarrow \quad \sqrt{5}q = p$

$\Rightarrow \quad 5q^2 = p^2$ $\left(\dfrac{1}{2}\text{mark}\right)$

$\Rightarrow \quad 5 \text{divides } p^2$

$\Rightarrow \quad 5 \text{divides } p \quad (1)$

So p =5a for some integer a $\left(\frac{1}{2}\text{mark}\right)$

Substituting $p = 5a$ in $5q^2 = p^2$

$$5q^2 = 25a^2$$

$\Rightarrow \quad q^2 = 5a^2$ $\left(\frac{1}{2}\text{mark}\right)$

$\Rightarrow \quad 5 \text{ divides } q^2$

$\Rightarrow \quad 5 \text{ divides } q \quad (2)$

From (1) & (2) 5 is a common factor to p and q which contradicts the fact that P and q are co prime

$\therefore$ Our assumption is wrong and hence $\sqrt{5}$ is irrational. $\left(\frac{1}{2}\text{mark}\right)$

Ans17 Let $A(x, y)$ be the required point which is at a distance of 5 units from the point P(0,5) and 3 units from Q(0,1)

So AP =5 and AQ = 3

$\Rightarrow \quad \sqrt{(x-0)^2 + (y-5)^2} = 5$ $\left(\frac{1}{2}\text{mark}\right)$

$\Rightarrow \quad (x-0)^2 + (y-5)^2 = 25$

$\Rightarrow \quad x^2 + y^2 - 10y = 0 \quad (1)$ $\left(\frac{1}{2}\text{mark}\right)$

$\sqrt{(x-0)^2 + (y-1)^2} = 3$ $\left(\frac{1}{2}\text{mark}\right)$

$x^2 + (y-1)^2 = 9$

$x^2 + y^2 - 2y - 8 = 0$ (2) $\left(\frac{1}{2}\text{mark}\right)$

Equation (1) – Equation (2) gives:

$-8y + 8 = 0 \Rightarrow y = 1$

Substituting $y=1$ in (1)

$$x^2-9=0 \Rightarrow x=\pm 3$$

$\therefore$ The required points are (3, 1) and (-3, 1) $\left(\frac{1}{2}\text{mark}\right)$

Ans18 $(\sin A+\operatorname{cos}ecA)^2+(\cos A+\sec A)^2$

$$=\sin^2 A+\text{cosec}^2 A+2\sin A\ \operatorname{cos}ec\, A+\cos^2 A$$

$$+\sec^2 A+2\cos A\sec A \quad \left(\frac{1}{2}\text{mark}\right)$$

$$=\left(\sin^2 A+\cos^2 A\right)+2+2+\text{cosec}^2 A+\sec^2$$

(Since sin A.cosec A=1 and cosA.secA =1) (1Mark)

$$=1+2+2+1+\cot^2 A+1+\tan^2 A \quad \text{(1 Mark)}$$

(Since, $\text{cosec}^2 A=1+\cot^2 A\ and\ \sec^2 A=1+\tan^2 A$)

$$=7+\cot^2 A+\tan^2 A$$

=RHS $\left(\frac{1}{2}\text{mark}\right)$

OR

$$(1+\cot\theta - \text{cosec}\,\theta)(1+\tan\theta+\sec\theta)$$

$$=\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right) \quad \left(\frac{1}{2}\text{mark}\right)$$

$$=\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right) \quad \left(\frac{1}{2}\text{mark}\right)$$

$$=\frac{\left(\sin\theta+\cos^2\theta\right)-(1)^2}{\sin\theta\ \cos\theta} \quad \left(\frac{1}{2}\text{mark}\right)$$

$$=\frac{\sin^2\theta+\cos^2\theta+2\ \sin\theta\ \cos\theta-1}{\sin\theta\ \cos\theta} \quad \left(\frac{1}{2}\text{mark}\right)$$

$$=\frac{1+2\sin\theta\cos\theta-1}{\text{siin}\cos\theta} \quad \left(\frac{1}{2}\text{mark}\right)$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$$ $\left(\frac{1}{2}\text{mark}\right)$

Ans19 Let $\frac{1}{x+y} = a, \quad \frac{1}{y-x} = b$

$10a + 4b = -2 \quad \rightarrow \quad (1)$

$15a - 7b = 10 \quad \rightarrow \quad (2)$ $\left(\frac{1}{2}\text{mark}\right)$

(1) × 3 and (2) × 2 gives

$\cancel{30}\, a + 12b = -6$

$\cancel{30}\, a - 14b = 20$

$26b = -26$

$\Rightarrow b = -1$ (1 mark)

Substituting b = -1 in (1):

$10A - 4 = -2$

$\Rightarrow 10a = 2$ $\left(\frac{1}{2}\text{mark}\right)$

$\Rightarrow a = \frac{1}{5}$

$\therefore \quad x + y = 5$

$-x + y = -1$

$2y = 4 \Rightarrow y = 2$

$\therefore \quad x = 3$ (1 mark)

OR

For real and distinct roots: $D > 0$ $\left(\frac{1}{2}\text{mark}\right)$

Discriminant D = b^2-4ac

$\left[-2(1+2m)\right]^2 - 4\,(2m)(3+2m) > 0$ $\left(\frac{1}{2}\text{mark}\right)$

$4(1+2m)^2 - 4\,(2m)(3+2m) > 0$

$1+\cancel{4m^2}+4m-6m-\cancel{4m^2}>0$ $\left(\frac{1}{2}\text{mark}\right)$

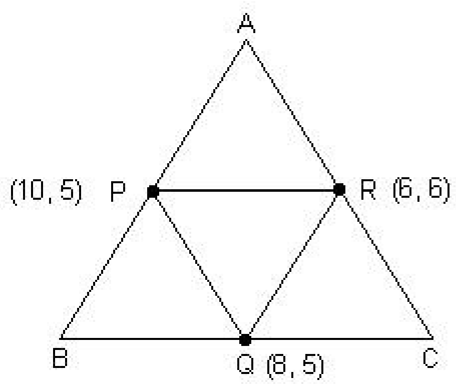
$1-2m>0$ $\left(\frac{1}{2}\text{mark}\right)$

$\Rightarrow \quad 1>2m$

$\Rightarrow \quad \frac{1}{2}>m$

$\Rightarrow \quad m<\frac{1}{2}$ $(1\,\text{mark})$

Ans20



(1 mark)

We know that area of triangle formed by joining the midpoint of sides of a triangle is $\frac{1}{4}$th the area of the triangle.

ar(ΔPQR) $=\frac{1}{4}$ (ar ΔABC) $\left(\frac{1}{2}\text{mark}\right)$

ar(ΔPQR) $=\frac{1}{2}\left[10(6-5)+6(5-5)+8(5-6)\right]$ (1 mark)

$=\frac{1}{2}[10-8]$

$=1 \text{ sq unit}$

So ar (ΔABC) = 4 sq unit $\left(\frac{1}{2}\text{mark}\right)$

Ans21 $3x^2 - 11x + 14$

$\alpha + \beta = \frac{11}{3}, \ \alpha\beta = \frac{14}{3}$ (1 mark)

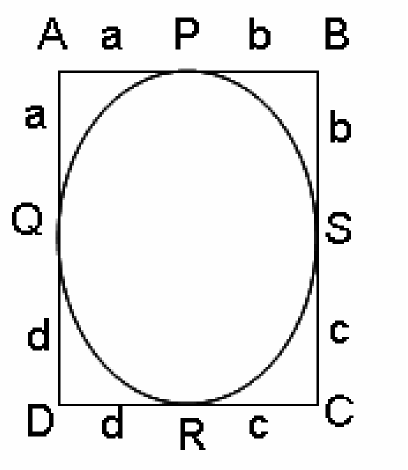
$\alpha + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ (1 mark)

$= \left(\frac{11}{3}\right)^2 - 2\left(\frac{14}{3}\right)$ $\left(\frac{1}{2}\text{mark}\right)$

$= \frac{121}{9} - \frac{28}{3}$

$= \frac{37}{9}$ $\left(\frac{1}{2}\text{mark}\right)$

Ans22 We know that tangents drawn from an external point are equal.



$\therefore$ Let $AP = AQ = a$

$BP = BS = b$

$CS = CR = C$ $\left(\frac{1}{2}\text{mark}\right)$

$DQ = DR = d$

Since ABCD is a parallelogram, opposite sides are equal.

$\not{a} + b = \not{c} + d$ $\left(\frac{1}{2}\text{mark}\right)$

$\underline{\not{a} + d = \not{c} + b}$

on subtracting , we get

$b-d = d-b$

$\Rightarrow \quad 2b = 2d$

$\Rightarrow \quad b = d$

$\therefore \quad AB = a+b$

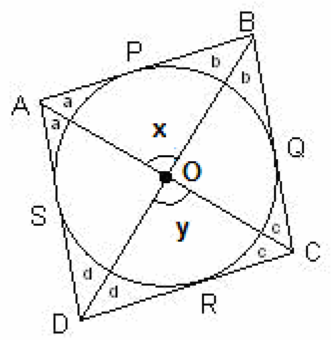
$= a+d$ $\left(\frac{1}{2}\text{mark}\right)$

$= AD$

Since adjacent sides are equal ABCD is a rhombus. $\left(\frac{1}{2}\text{mark}\right)$

OR

We know that the two tangents drawn from an external point are equally inclined to the line joining the point and centre $\left(\frac{1}{2}\text{mark}\right)$



$\left(\frac{1}{2}\text{mark}\right)$

$\therefore \quad$ Let $\angle OAP = \angle OAS = a \quad \angle OCQ = \angle OCR = c$ $\left(\frac{1}{2}\text{mark}\right)$

$\angle OBP = \angle OBQ = b \quad \angle ODR = \angle ODS = d$

In $\Delta AOB : a+b+x = 180^0$ $\left(\frac{1}{2}\text{mark}\right)$

In $\Delta COD : c+d+y = 180^0$

On adding $a+b+c+d+x+y = 360$ $\left(\frac{1}{2}\text{mark}\right)$

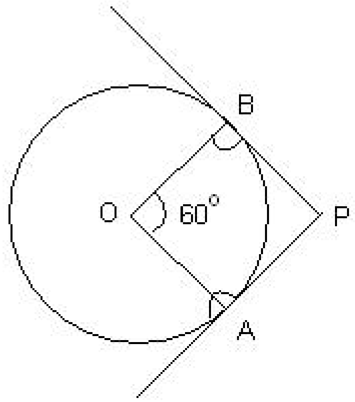
$\Rightarrow \quad 180+x+y = 360$

(Using angle sum property of quadrilateral 2a+2b+2c+2d=360)

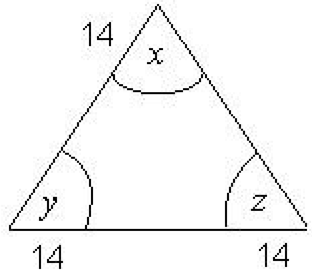
So x + y = 180° $\left(\frac{1}{2}\text{mark}\right)$

Hence proved.

Ans23 Construction of circle and 2 radii OA,OB at an angle of 60° (1 mark)
Construction of tangents through the points on the circle (2 marks)



Ans24 Let the angles of triangle be x, y, z.



Area grazed by the three horses

$$=\frac{x}{360}\pi r^2+\frac{y}{360}\pi r^2+\frac{z}{360}\pi r^2$$ (1 mark)

$$=\frac{\pi r^2}{360}(x+y+z)$$ $\left(\frac{1}{2}\text{mark}\right)$

$= \frac{\pi r^2}{360} \times 180$ $\left(\frac{1}{2}\text{mark}\right)$

$= \frac{22}{\not{7}} \times \not{14} \times 14 \times \frac{1}{\not{2}}$ $\left(\frac{1}{2}\text{mark}\right)$

$= 308\ m^2$ $\left(\frac{1}{2}\text{mark}\right)$

Ans25 $a_{46} = 25$

$\Rightarrow a + 45d = 25$ $\left(\frac{1}{2}\text{mark}\right)$

$S_{91} = \frac{91}{2}[2a + 90d]$ (1mark)

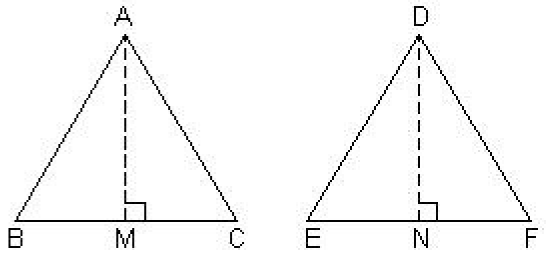
$= 91(a + 45d)$ $\left(\frac{1}{2}\text{mark}\right)$

$= 91 \times 25$ $\left(\frac{1}{2}\text{mark}\right)$

$= 2275$ $\left(\frac{1}{2}\text{mark}\right)$

Section D

Ans26 Given: $\Delta\ ABC \sim \Delta\ DEF$



To Prove: $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

Construction: Draw $AM \perp BC$ and $DN \perp EF$

Proof: In ΔABC and ΔDEF (1mark)

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{\frac{1}{2}\times BC\times AM}{\frac{1}{2}\times EF\times DN} = \frac{BC}{EF}.\frac{AM}{DN} \qquad ...(i)$$

$$\left[\text{Area of } \Delta = \frac{1}{2}\times \text{base}\times\text{corresponding altitude}\right]$$

$\therefore \quad \Delta ABC \sim \Delta DEF \qquad ...(\text{Given})$

$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} \qquad ...(\text{Sides are proportional})...(ii)$

$\angle B = \angle E \qquad ...(\because \Delta ABC \sim \Delta DEF)$

$\angle M = \angle N \qquad ...(\text{each } 90^\circ)$

$\therefore \quad \Delta ABM \sim \Delta DEN \qquad ...(\text{AA Similarity})$

$\therefore \quad \frac{AB}{DE} = \frac{AM}{DN} \qquad ...(iii)[\text{Sides are proportional}]$

From (ii) and (iii), we have

$\frac{BC}{DE} = \frac{AM}{DN}$ (1 mark)

From (i) and (iv), we have

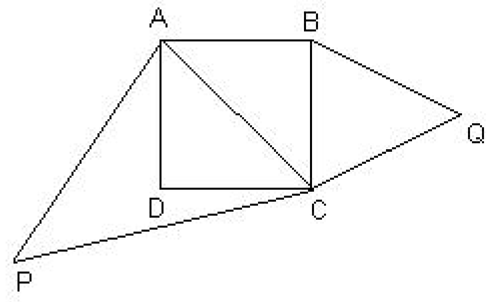
$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC}{EF}.\frac{BC}{EF} = \frac{BC^2}{EF^2}$$

Similarly, we can prove that

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}$$

(2 marks)

$$\therefore \quad \mathbf{\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}}$$



ΔBCQ and ΔACP are equilateral triangles and therefore similar.

(1 mark)

$AC^2 = AB^2 + BC^2 = 2BC^2$ (By Pythagoras theorem) $\left(\frac{1}{2}\text{mark}\right)$

Using the above theorem

$$\frac{\text{area } \triangle ACP}{\text{area } \triangle BCQ} = \frac{AC^2}{BC^2} = \frac{2BC^2}{BC^2} = 2$$ $\left(\frac{1}{2}\text{mark}\right)$

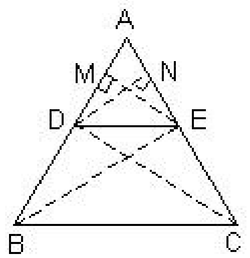
OR

Statement: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. (1 Mark)

Given: In $\Delta ABC, DE \parallel BC$

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Draw $EM \perp AD$ and $DN \perp AE$. Join B to E and C to D



(1 mark)

Proof: In ΔADE and ΔBDE

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times DB \times EM} = \frac{AD}{DB} \qquad ...(i)$$

[Area of $\Delta = \frac{1}{2} \times \text{base} \times$ corresponding altitude]

In ΔADE and ΔCDE

$$\frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times EC \times DN} = \frac{AE}{EC} \qquad ...(ii)$$

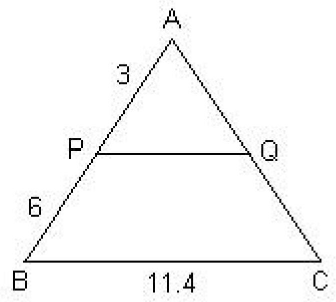
$\because \quad DE \parallel BC \qquad ...(\text{Given})$

$\therefore \quad ar(\Delta BDE) = ar(\Delta CDE) \qquad ...(iii)$

($\because$ Δs on the same base and between the same parallel sides are equal in area)

From (i), (ii) and (iii)

$$\frac{\mathbf{AD}}{\mathbf{DB}} = \frac{\mathbf{AE}}{\mathbf{EC}}$$ (2 marks)



Since $PQ \parallel BC$

$\Delta APQ \sim \Delta ABC$ (By AA condition) (1 mark)

$$\therefore \quad \frac{AP}{AB} = \frac{PQ}{BC}$$

$$\Rightarrow \quad \frac{3}{9} = \frac{PQ}{11.4}$$ (1 mark)

$$\Rightarrow \quad PQ = \frac{34.2}{9} = 3.8\,cm$$

Ans27 Let length of rectangle = x m

Breadth = y m

Area $= xy\ m^2$. $\left(\frac{1}{2}\text{mark}\right)$

$(x+7)(y-3) = xy$ (1 mark)

$\Rightarrow\ -3x+7y-21=0\ \rightarrow (1)$ $\left(\frac{1}{2}\text{mark}\right)$

$(x-7)(y+5) = xy$ (1 mark)

$5x-7y-35=0\ \rightarrow\ (2)$ $\left(\frac{1}{2}\text{mark}\right)$

(1) + (2) gives:

$2x-56=0$

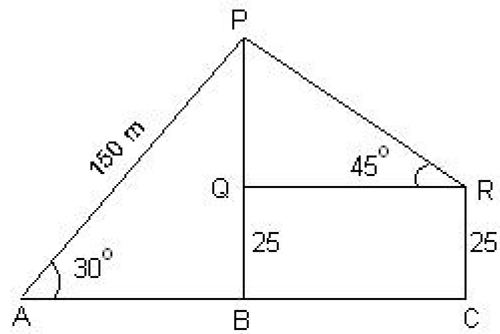
$\Rightarrow\ x = 28m$ (1 mark)

On substituting x = 28 m in equation (2), we get y = 15 m

The length is 28 m and the breadth is 15m. $\left(\frac{1}{2}\text{mark}\right)$

Therefore, area is $420m^2$ (1 mark)

Ans28 A and R are the positions of the two boys. P is the point where the two kites meet. $\left(\frac{1}{2}\text{mark}\right)$



(1 mark)

In Δ ABP

$$\sin 30^o = \frac{PB}{AP}$$

$$\frac{1}{2} = \frac{PB}{150}$$ $\left(1\frac{1}{2}\text{mark}\right)$

$$\Rightarrow \quad PB = 75m$$

$$\text{and} \quad QB = 25m$$

$$\Rightarrow PQ = 50m$$ (1 mark)

In Δ PQR

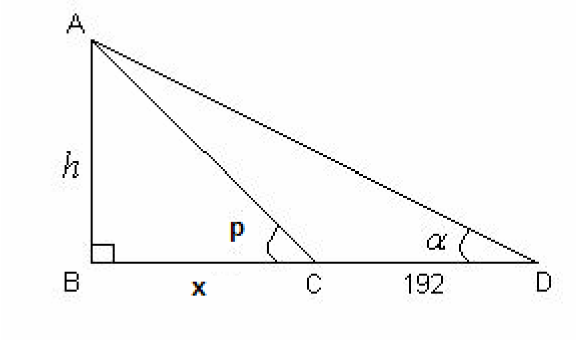
$$\sin 45^o = \frac{PQ}{PR}$$

$$\frac{1}{\sqrt{2}} = \frac{50}{PR}$$ $\left(1\frac{1}{2}\text{mark}\right)$

$$\Rightarrow \quad 50\sqrt{2} = PR$$

$\therefore$ The boy should have a string of length 70.7m $\left(\frac{1}{2}\text{mark}\right)$

OR



(1 mark)

D is the initial point of observation and C is the next point of observation.

AB is the tower of height h. Let BC = x

$$\tan\alpha = \frac{AB}{BD}$$

$$\frac{5}{12} = \frac{h}{x+192} \qquad \left(\frac{1}{2}\text{mark}\right)$$

$$\Rightarrow \; 12h - 5x - 960 = 0 \quad \rightarrow (1) \qquad \text{(1 mark)}$$

$$\tan\ p = \frac{AB}{BC}$$

$$\frac{3}{4} = \frac{h}{x} \qquad \left(\frac{1}{2}\text{mark}\right)$$

$$\Rightarrow \; 3x = 4h \rightarrow (2) \qquad \text{(1 mark)}$$

From (2): $12h = 9x$ and substituting in (1):

$$9x - 5x = 960$$

$$4x = 960 \qquad \text{(1 mark)}$$

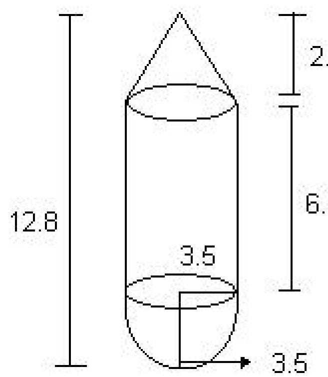
$$\Rightarrow \quad x = 240$$

$\therefore h = \frac{3 \times 240}{4}$ from (2) $\left(\frac{1}{2}\text{mark}\right)$

$= 180$

$\therefore$ The height of the tower is 180 m. $\left(\frac{1}{2}\text{mark}\right)$

Ans29



Height of cone $= 12.8 - (6.5 + 3.5)$

= 2.8 c (1 mark)

Slant height $l = \sqrt{(3.5)^2 + (2.8)^2}$

$= \sqrt{12.25 + 7.84}$

$= \sqrt{20.09}$

$= 4.48$ $\left(1\frac{1}{2}\text{mark}\right)$

$TSA = 2\pi r^2 + 2\pi rh + \pi rl$

$= \pi r(2r + 2h + l)$

$= \frac{22}{7} \times \frac{7}{2}(7 + 13 + 4.48)$ (1 mark)

$= 11 \times 24.48$

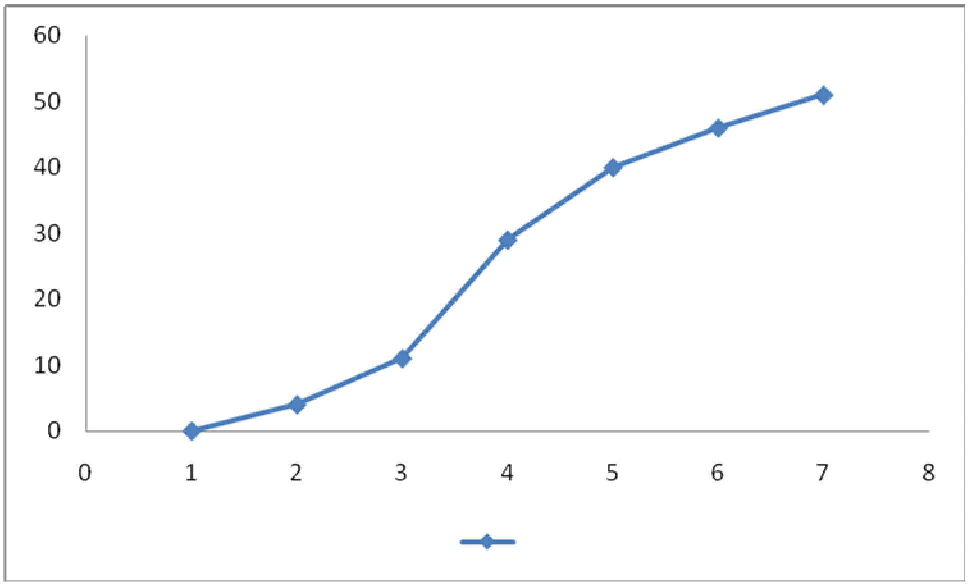
$= 269.28$ $\left(1\frac{1}{2}\text{mark}\right)$

$\therefore$ The surface area of the solid is 269.28 cm^2 $\left(\frac{1}{2}\text{mark}\right)$

Ans30

CI	f	$C.f$
Less than 140	4	4
140 - 145	7	11
145 - 150	18	29
150 – 155	11	40
155 – 160	6	46
160 – 165	5	51
	51	

(2 marks)



(2 marks)

$n = 51 \Rightarrow \frac{n}{2} = 25.5$

Median class $= 145 - 150$

Median $= l + \left(\frac{\frac{n}{2} - cf}{f} \right) h$

$$= 145 + \left(\frac{25.5 - 11}{18} \right) 5$$

$$= 145 + \frac{14.5 \times 5}{18}$$ (2 marks)

$$= 145 + \frac{72.5}{18}$$

$$= 149.02$$