Sample paper -2010

Class : XII
Sub : **MATHEMATICS**

Time allowed: 3 hrs M.Marks:100

General Instructions:

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into three sections – A, B and C. Section A comprises of 10 questions of 1 mark each; Section B comprises of 12 questions of 4 marks each and Section C comprises of 7 questions of 6 marks each.

(iii) Use of calculator is not permitted. You may ask for logarithmic tables if required.

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SECTION - A

1. Construct a 2x2 matrix , $A=\left[a_{ij}\right]$, whose elements are given by $a_{ij}=\frac{(i-2j)^2}{3}$.
2. Find the values of x , y and z where $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix}=\begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$
3. Evaluate : $\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$
4. Find k if $f(x)=\begin{cases} kx^2 & , x\neq 0 \\ 5 & , x=2 \end{cases}$ is continuous at x = 2.
5. The total revenue from the sale of x units of a product is given by $R(x)=6x^2+13x+10$. Find the marginal revenue when x =10 .
6. Evaluate : $\int \frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}dx$
7. Evaluate : $\int_0^1 \frac{2x}{1+x^2}dx$
8. Find the distance of the point $(2,3,4)$ from the plane $\vec{r}.\left(3\hat{i}-6\hat{j}+2\hat{k}\right)=-11$.
9. If $\vec{a}$ is a unit vector and $\left(\vec{x}+\vec{a}\right)\left(\vec{x}-\vec{a}\right)=15$, find $\left|\vec{x}\right|$.
10. Evaluate : $\sin^{-1}\left(\sin\frac{5\pi}{4}\right)$

SECTION – B

.11.Prove that $\cot^{-1}\frac{ab+1}{a-b}+\cot^{-1}\frac{bc+1}{b-c}+\cot^{-1}\frac{ca+1}{c-a}=0$

12. If a , b , c are the pth , qth , rth terms of a G.P. , prove that $\begin{vmatrix}\log a & p & 1\\ \log b & q & 1\\ \log c & r & 1\end{vmatrix}=0$.

OR,

Without expanding prove that $\begin{vmatrix}x+y & x & x\\ 5x+4y & 4x & 2x\\ 10x+8y & 8x & 3x\end{vmatrix}=x^3$

13. Differentiate $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t. $\sqrt{1-x^2}$.

14. Show that the relation R in the set A=$\{1,2,3,4,5\}$ given by R=$\{(a,b):|a-b| \text{ is even}\}$ is an equivalence relation. Show that all the elements of $\{1,3,5\}$ are related to each other and all the elements of $\{2,4\}$ related to each other. But no element of $\{1,3,5\}$ is not related to any element of $\{2,4\}$.

15. If y = tan x + sec x , prove that $\frac{d^2y}{dx^2}=\frac{\cos x}{(1-\sin x)^2}$.

16. Evaluate : $\int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx$.

17. Evaluate : $\int_1^4 f(x)dx$ where $f(x)=\begin{cases}2x+8 & ,1\le x\le 2\\ 6x & ,2\le x\le 4\end{cases}$.

OR,

Evaluate : $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{1}{1+\sqrt{\tan x}}dx$

18. Find the direction cosines of two lines which are connected by the relations $l-5m+3n=0$, $7l^2+5m^2-3n^2=0$

19. Find the perpendicular distance of the point $(2,3,4)$ from the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$.

20. Solve the differential equation $\frac{dy}{dx}+y\cot x=x^2\cot x+2x$.

OR,

Solve the differential equation $\frac{dy}{dx}-\frac{y}{x}+\text{cosec}\frac{y}{x}=0$; y = 0 when x = 1

21. A girl throws a die . If she gets 5 or 6 , she tosses a coin three times ,otherwise she tosses a coin once . If she obtained exactly one head , what is the probability that she threw 1,2,3 or 4 with the die .

22. Find the equation of the tangent to the curve $y=\sqrt{3x-2}$, which is parallel to the line $4x-2y+5=0$.

SECTION – C

23. Find the product of matrices $A=\begin{bmatrix}-5 & 1 & 3\\ 7 & 1 & -5\\ 1 & -1 & 1\end{bmatrix}$., $B=\begin{bmatrix}1 & 1 & 2\\ 3 & 2 & 1\\ 2 & 1 & 3\end{bmatrix}$ and use it to solve the equations

$x+y+2z=1, 3x+2y+z=7, 2x+y+3z=2$

24. If length of three sides of trapezium other than base are equal to 10 cm., then find the area of the trapezium when it is maximum.

OR,

Prove that radius of a right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

25. A manufacturing company makes two models A and B of a product. Each piece of model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of model B requires 12 labour hours for fabricating and 3 labour hour for finishing. For fabricating and finishing the maximum labour are available are 180 and 30 respectively. The company makes a profit of Rs.8000 on each piece of model A and model B should be manufactured per week to realize a maximum profit . What is the maximum profit per week.

26. Find the area of the region $\{(x,y): y^2 \le 4x, 4x^2+4y^2 \le 9\}$.

27. An urn contains 25 balls of which 10 balls bears a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn ,its mark is noted down and is replaced. If 6 balls are drawn in this way , find the probability that
    (i) all will bear 'X' mark .
    (ii) not more than 2 will bear 'Y' mark.
    (iii) at least one ball will bear 'Y' mark
    (iv) the no. of balls with 'X' mark and 'Y' mark will be equal.

28. Integrate : $\int \frac{dx}{x^4+3x^2+1}$

29. Find the length and the equation of the line of shortest distance between the lines $\frac{x-1}{1}=\frac{y+7}{3}=\frac{z+2}{2}$ and $\frac{x-3}{-1}=\frac{y-4}{2}=\frac{z+2}{1}$.

Submitted by

Mrinal Sarma

PGT,Gurukul Grammar Senior Secondary School ,Guwahati, Assam

Ph. 09864066569