



Code No. **Series AG-TM-1**

CLASS XII

TMG-D/79/89

- Please check that this question paper contains 3 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 29 questions.

General Instructions: -

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A contains 10 questions of 1 marks each, Section B is of 12 questions of 4 marks each and Section C is of 7 questions of 6 marks each.
3. Write the serial number of the question before attempting it.
4. If you wish to answer any question already answered, cancel the previous answer.
5. In questions where internal choices is provided. You must attempt only one choice.

MATHEMATICS

Time Allowed : 3 hours

Maximum Marks : 100

PART – A

1. If $f : R \rightarrow R$ be given by $f(x) = (3 - x^3)^{1/3}$. then find the value of $fof(x)$.
2. If A is a matrix of order 2×3 and B is a matrix of order 3×5 , what is the order of matrix $(AB)'$.
3. Given an example to show that the relation R in the set of natural numbers defined by $R = \{ (x, y) : x, y \in N, x \leq y^2 \}$ is not transitive.
4. Find the value of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$.
5. Find x, if $\tan^{-1} 4 + \cot^{-1} x = \frac{\pi}{2}$.
6. Find f(x) satisfying the following : $\int (2x+1)\sqrt{x^2+x+1}dx = f(x) + c$.
7. Evaluate : $\int_0^{2/3} \frac{dx}{4+9x^2}$.
8. Write a vector normal to the plane $\vec{r} = \mu\vec{a} + \lambda\vec{b}$.
9. Write the value of k for which the line $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-1}{k}$ is perpendicular to the normal to the plane $\vec{r} \cdot (2i + 3j + 4k) + 4 = 0$.
10. Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

PART – B

11. Let $f, g : R \rightarrow R$ be defined as $f(x)=|x|$ and $g(x)= [x]$ where $[x]$ denotes the greatest integer less than or equal to x . Find $f \circ g \left(\frac{5}{2} \right)$ and $g \circ f \left(-\sqrt{2} \right)$.

12. Prove that : $2 \tan^{-1} \left(\frac{1}{2} \right) + \tan^{-1} \left(\frac{1}{7} \right) = \tan^{-1} \left(\frac{31}{17} \right)$

13. If $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$, show that $A^2 - 6A + 17I = 0$ Hence, find A^{-1} .

OR

If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$, where n is any positive integer.

14. For what value of a and b , the function f defined as: $f(x) = \begin{cases} 3ax + b, & \text{if, } x < 1 \\ 11, & \text{if, } x = 1 \\ 5ax - 2b, & \text{if, } x > 1 \end{cases}$ is

continuous at $x = 1$.

15. If $y = (\log x)^{\cos x} + \frac{x^2 + 1}{x^2 - 1}$, find $\frac{dy}{dx}$

16. For the curve $y = 4x^3 - 2x^5$ find all the points at which tangent passes through the origin.

17. Using properties of definite integral, evaluate: $\int_0^\pi \frac{x dx}{4 - \cos^2 x}$.

18. Solve the following differential equation : $xy \frac{dy}{dx} + \sqrt{1 + x^2 + y^2 + x^2 y^2} = 0$.

OR

Find the particular solution of the differential equation

$(x dy - y dx) y \sin \left(\frac{y}{x} \right) = (y dx + x dy) x \cos \frac{y}{x}$, given that $y = \pi$ when $x = 3$.

19. Form the differential equation of the family of circles having radii 3.

20. Find a vector of magnitude 5 units perpendicular to each of the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

OR

If the sum of the two unit vector is unit vector prove that magnitude of their difference is $\sqrt{3}$.

21. Find the distance of the point $(-2, 3, -4)$ from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$

OR

Find the equation of the plane passing through the point $(1, 1, 1)$ and containing the line $\vec{r} = -3\hat{i} + \hat{j} + 5\hat{k} + \lambda(3\hat{i} - \hat{j} - 5\hat{k})$. Also prove that the plane contain the line $\vec{r} = -\hat{i} + 2\hat{j} + 5\hat{k} + \mu(\hat{i} - 2\hat{j} - 5\hat{k})$

22. In an examination, 6 questions of true- false type are asked. A student tosses a fair die to determine his answer to each question. If the die show 1 or 2 , he answers true otherwise he answers false. Find the probability that he answers at most 4 questions correctly it.

PART – C

23. Using properties of determinants, show that :

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3.$$

24. Show that the semi-vertical angle of a right circular cone of maximum volume and of given slant height is $\tan^{-1} \sqrt{2}$

OR

Given the sum of the perimeter of a square and a circle, show that the sum of their areas is least when the side of the square is equal to diameter of the circle.

25. Find the area of the region : $\{(x, y) : y^2 \geq 6x, x^2 + y^2 \leq 16\}$.

26. Evaluate $\int_1^3 (x^2 + 5x) dx$ as the limit of sums.

27. Find the coordinates of the image of the point (1,3,4) in the plane $2x - y + z + 3 = 0$.

OR

Find the distance between the point P(6,5,9) and the plane determined by the points A (3,-1,2) B (5,2,4) and C (-1,-1,6).

28. A man has Rs 1,500 for purchasing rice and wheat. A bag of rice and a bag of wheat cost Rs 180 and Rs 120 respectively. He has the storage capacity of at most 10 bags. He earns a profit of Rs 11 and Rs 9 per bag of rice and wheat respectively. Formulate the above problem as an LPP to maximize the profit and solve it graphically.

29. A doctor is to visit a patient. From the past experience it is known that the probabilities of the doctor coming by train, bus, scooter or taxi are $\frac{1}{10}, \frac{1}{5}, \frac{3}{10}$, and $\frac{2}{5}$ respectively. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$ if he comes by train, bus or scooter respectively, but by taxi he will not be late. When he arrives he is late. What is the probability that he comes by bus ?
