



Code No. **Series AG-TM-2**

CLASS XII

TMG-D/79/89

- Please check that this question paper contains 3 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 29 questions.

General Instructions: -

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A contains 10 questions of 1 marks each, Section B is of 12 questions of 4 marks each and Section C is of 7 questions of 6 marks each.
3. Write the serial number of the question before attempting it.
4. If you wish to answer any question already answered, cancel the previous answer.
5. In questions where internal choices is provided. You must attempt only one choice.

MATHEMATICS

Time Allowed : 3 hours

Maximum Marks : 100

PART – A

1. Find the unit vector in the direction of the vector $\vec{a} = 3\hat{i} + 6\hat{j} - 2\hat{k}$. Find the direction cosine of vector.
2. Find the value of the following : $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$.
3. Let \vec{a} and \vec{b} be two vectors such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector. Then what is the angle between \vec{a} and \vec{b} ?
4. If $\begin{bmatrix} x+2y & -y \\ 3x & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 6 & 4 \end{bmatrix}$, find the value of x and y.
5. If $\begin{vmatrix} x & x \\ 1 & x \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$, find the value of x.
6. Find the principal value of $\tan^{-1}(-1)$
7. Evaluate : $\int \frac{1}{\sqrt{1-x^2} \sin^{-1} x} dx$.
8. If $\int_0^a 3x^2 dx = 8$, find the value of a.
9. Let * be a binary operation on the set Q of rational number given as $a*b = (2a-b)^2$, $a, b \in Q$ Find $3*5$.
10. If A is a matrix of order 3×3 and $|A| = 8$, then write the value of $|adj.A|$.

PART – B

11. Three vectors \vec{a}, \vec{b} and \vec{c} satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$, if $|\vec{a}| = 1, |\vec{b}| = 4$ and $|\vec{c}| = 2$

12. Write into the simplest form : $\cot^{-1}(\sqrt{1+x^2} - x)$.
OR

Solve : $\tan^{-1} \frac{x-1}{x-2} + \tan^{-1} \frac{x+1}{x+2} = \frac{\pi}{4}$.

13. Find the shortest distance between the lines l_1 and l_2 whose vector equation are given below:

$$l_1 : \vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$$

$$l_2 : \vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$$

14. Solve the following differential equation:

$$\cos^2 x \frac{dy}{dx} + y = \tan x$$

15. Find the particular solution, satisfying the given condition, for the following differential equation :

$$x \frac{dy}{dx} \cdot \sin\left(\frac{y}{x}\right) + x - y \sin\left(\frac{y}{x}\right) = 0, \quad y(1) = \frac{\pi}{2}$$

16. Using properties of determinants show that if x,y and z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then $1 + xyz = 0$.

17. Two cards are drawn successively with replacement from a well shuffled deck of 52 cards. Find the probability distribution of number of aces

18. If $x = a \left\{ \cos t + \log \tan \frac{t}{2} \right\}$ and $y = a \sin t$, find $\frac{dy}{dx}$.

19. Evaluate : $\int x \sin^{-1} x dx$.

OR

Evaluate : $\int \frac{\sin x}{(1 - \cos x)(2 - \cos x)} dx$

20. Let $A = \mathbb{R} - \{2\}$ and $B = \mathbb{R} - \{1\}$, if $f : A \rightarrow B$ is a mapping defined by $f(x) = \frac{x-1}{x-2}$, show that f is bijective.

21. If $y = x^{\cot x} + (\sin x)^x$, find $\frac{dy}{dx}$.

OR

If $y = \sin(\log x)$, prove that $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$

22. Find the intervals in which the following function $f(x)$ is (a) increasing (b) decreasing:

$$f(x) = 2x^3 - 9x^2 + 12x + 15$$

OR

Verify Rolle's theorem for the function: $f(x) = x^2 - 5x + 4$ on $[1, 4]$.

PART – C

23. Show that the height of the closed right circular cylinder, of given volume and minimum total surface area is equal to its diameter.

OR

Find the point on the curve $x^2 = 8y$ which is nearest to the point (2,4).

24. If a young man rides his motorcycle at 25km/hour, he had to spend Rs 2 per km on petrol. If he rides at a faster speed of 40km/hour, the petrol cost increases at Rs 5 per km. He has Rs 100 to spend on petrol and wishes to find what is maximum distance he can travel within one hour. Express this as an LPP and solve it graphically.
25. A man is known to speak the truth 3 out of 4 times. He throws a dice and reports that it is a six. Find the probability that it is actually a six.
26. Using matrices, solve the following systems of equations: $x + 2y + z = 7$; $x + 3z = 11$; $2x - 3y = 1$.

27. Using properties of definite integral, prove the following: $\int_0^{\pi} \frac{x \tan x}{\sec x \cos ex} dx = \frac{\pi^2}{4}$

OR

Evaluate : $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$

28. Find the area of the region bounded by the parabola $x^2 = 4y$ and the line $x = 4y - 2$.
29. Find the equation of the plane passing through the point (-1,-1,2) and perpendicular to each of the following planes: $2x + 3y - 3z = 2$ and $5x - 4y + z = 6$.
