Year of Quality education 18th

Code No. **Series AG-6** CLASS XII **TMG -D/79/89**

- Please check that this question paper contains 3 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 29 questions.

General Instructions: -

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A contains 10 questions of 1 marks each, Section B is of 12 questions of 4 marks each and Section C is of 7 questions of 6 marks each.
3. Write the serial number of the question before attempting it.
4. If you wish to answer any question already answered, cancel the previous answer.
5. In questions where internal choices is provided. You must attempt only one choice.

MATHEMATICS

Time Allowed : 3 hours **Maximum Marks : 100**

PART – A

1. If a binary operation $\oplus$ is defined by $a \oplus b = 2a - 3b$, find 8 $\oplus$ 3.
2. Find the value of $\cos\left[\sin^{-1}\left(\frac{1}{7}\right) + \cos^{-1}\left(\frac{1}{7}\right)\right]$.
3. If a matrix has 8 elements, what are the possible orders it can have?
4. Evaluate the determinant $\begin{vmatrix} 1 & 2 & 3 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$.
5. For the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, find M_{12} and C_{23} where M_{12} is minor of the element in first row and second column and C_{23} is cofactor of the element in second row and third column.
6. Find the derivative of $\cos^{-1}(\sin x)$ w.r.t. x.
7. Evaluate: $\int \left(x + \frac{1}{x}\right)^2 dx$.
8. Find the value of x, y and z so that the vectors $\vec{a} = 2x\hat{i} + 3\hat{j} + z\hat{k}$ and $\vec{b} = 2\hat{i} + y\hat{j} + z\hat{k}$ are equal.
9. If $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ find $\vec{a} \cdot \vec{b}$.
10. Find the direction cosines of a line passing through the point (-1, 0, 2) and (3, 4, 6).

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PART – B

11. Let A=R-{2} and B=R-{1}.if $f : A \to B$ is a mapping defined by $f(x) = \frac{x-1}{x-2}$, show that f is a bijection.

12. Solve the equation $\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}, x > 0$.

OR

Prove that $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right] = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(x^2\right)$.

13. Using the properties of determinants, show that : $\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = \left(a^3-1\right)^2$

14. If $y = (\cos x)^{\log x} + (\log x)^x$, find $\frac{dy}{dx}$.

15. For the value of a and b so that the function $f(x) = \begin{cases} ax^2 + b & , x < 2 \\ 2 & , x = 2 \\ 2ax - b & , x > 2 \end{cases}$ may be continuous.

16. Find the equation of the tangent to the curve $y = \cos 2t,\ \ x = \sin 3t$ at $t = \frac{\pi}{4}$.

17. Evaluate : $\int_0^{\frac{\pi}{2}} \log \sin x dx.$

18. Solve the differential equation : $y(1+e^x)dy = (y+1)e^x dx$.

OR

Solve the differential equation : $x\frac{dy}{dx} = y + \sqrt{x^2+y^2}$.

19. Solve the differential equation : $x\frac{dy}{dx} + y = x\cos x + \sin x$ given that $y\left(\frac{\pi}{2}\right) = 1$.

20. Find the unit vector perpendicular to the plane ABC where position vectors of points A, B and C are $2\hat{i} - \hat{j} + \hat{k}, \hat{i} + \hat{j} + 2\hat{k}$ and $2\hat{i} + 3\hat{k}$ respectively.

21. A variable plane which remains at a constant distance of 9 units from the origin, cuts the coordinate axes at the point A, B and C. show that the locus of the centroid of ΔABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{9}$.

OR

Find the foot of perpendicular to the point (2, 3, 4) to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also find the length of the perpendicular segment.

22. A car manufacturing factory has two plants. Plants P manufactures 70% of cars and plant Q manufactures 30%. At plant P, 80% of cars are rated of standard quality and at plant

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Q, 90% of cars rated of standard quality. A car is picked up at random and is found to be of standard quality. What is the probability that it has come from plant P?

OR

If X follows binomial distribution with mean 3 and variance $\frac{3}{2}$, find $P(X \le 5)$.

PART – C

23. Evaluate : $\int (3x-2)\sqrt{x^2+x+1}\,dx$.

24. Using integration, find the area of the triangular region whose vertices are P(1, 0), Q(2, 2) and R(3, 1).

OR

Evaluate: $\int_1^4 (x^2 - x)dx$ as limit of a sum.

25. Find the volume of the largest cone that can be inscribed in a sphere of radius a cm.

OR

Find the point of local maxima/minima for the function $f(x) = \sin 2x - x, -\frac{\pi}{2} \le x \le \frac{\pi}{2}$.
Also find the local maximum and local minimum values.

26. 3 bad eggs are mixed with 7 good ones. 3 eggs are taken at random from the lot. Find the probability distribution of number of bad eggs drawn. Find also the mean and variance of the probability distribution.

27. A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of the first machine is 12 hours and that of second machine is 9 hours. Each unit of product A required 3 hours on both machines and each unit of product B requires 2 hours on first machine and 1 hour on second machine. Each unit of product A is sold at a profit of Rs 5 and that of B at a profit of Rs 6. Find the production level for maximum profit graphically.

28. Determine whether or not the following pair of lines intersect. If these intersect, find the point of intersection, otherwise obtain the distance between them:

$$\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(3\hat{i} - \hat{j}) \;:\; \vec{r} = 4\hat{i} - \hat{k} + \mu(2\hat{i} + 3\hat{k}) .$$

29. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, find A^{-1} and use it to solve the system of equations : $x + y + 2z = 0$; $x + 2y - z = 9$; $x - 3y + 3z = -14$.

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