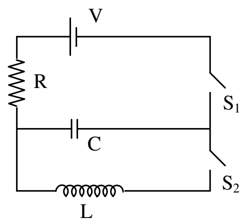
IITJEE 2013

PART C – PHYSICS

61. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and $\tau = RC$ is capacitive time constant). Which of the following statement is correct?



(1) At $t = \tau$, $q = CV/2$
(2) At $t = 2\tau$, $q = CV(1 - e^{-2})$
(3) At $t = \frac{\tau}{2}$, $q = CV(1-e^{-1})$
(4) Work done by the battery is half of the energy dissipated in the resistor.

Sol. **(2)**
Charge on the capacitor at any time 't' is
$q = CV\left(1-e^{-t/\tau}\right)$
at $t = 2\tau$
$q = CV\left(1-e^{-2}\right)$

62. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 pico farad in parallel with a load resistance 100 kilo ohm. Find the maximum modulated frequency which could be detected by it.

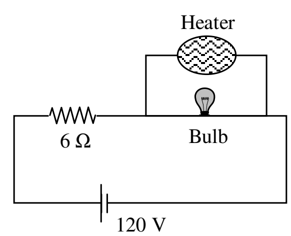
(1) 10.62 kHz (2) 5.31 MHz
(3) 5.31 kHz (4) 10.62 MHz

Sol. **(3)**
$$f_C = \frac{1}{2\pi RC} = \frac{1}{2\times3.14\times100\times10^3\times250\times10^{-12}} = 6.37\ \text{kHz}$$
f_C = cut off frequency
As we know that f_m □ f_C
$\therefore$ (3) is correct
Note: The maximum frequency of modulation must be less than f_m, where
$$f_m = f_C\frac{\sqrt{1-m^2}}{m}$$
$m \Rightarrow$ modulation index

63. The supply voltage to a room is 120 V. The resistance of the lead wires is 6 Ω. A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb?

(1) 2.9 Volt (2) 13.3 Volt
(3) 10.04 Volt (4) zero volt

Sol. **(3)**



Resistance of bulb = $\frac{120\times120}{60} = 240\Omega$

Resistance of Heater = $\frac{120\times120}{240} = 60\Omega$

Voltage across bulb before heater is switched on, $V_1 = \frac{120}{246} \times 240$

Voltage across bulb after heater is switched on, $V_2 = \frac{120}{54} \times 48$

Decrease in the voltage is $V_1 - V_2 = 10.04$ (approximately)
Note: Here supply voltage is taken as rated voltage.

64. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is:

(1) $\frac{Mg}{k}\left(1 - \frac{LA\sigma}{M}\right)$ (2) $\frac{Mg}{k}\left(1 - \frac{LA\sigma}{2M}\right)$

(3) $\frac{Mg}{k}\left(1 + \frac{LA\sigma}{M}\right)$ (4) $\frac{Mg}{k}$

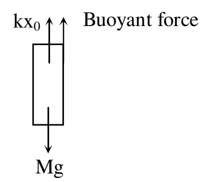
(Here k is spring constant)

Sol. **(2)**

At equilibrium $\Sigma F = 0$

$$kx_0 + \left(\frac{AL}{2}\sigma g\right) - Mg = 0$$

$$x_0 = Mg\left[1 - \frac{LA\sigma}{2M}\right]$$



65. Two charges, each equal to q, are kept at x = −a and x = a on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement ($y \ll a$) along the y-axis, the net force acting on the particle is proportional to:

(1) $-y$ (2) $\frac{1}{y}$

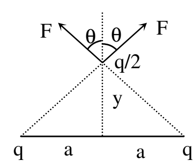
(3) $-\frac{1}{y}$ (4) y

Sol. **(4)**

$$F_{net} = 2F\cos\theta$$

$$= 2\frac{k \cdot q \cdot q/2}{\left(\sqrt{a^2 + y^2}\right)^2} \cdot \frac{y}{\sqrt{a^2 + y^2}}$$

$$= \frac{kq^2 y}{a^3} \quad (y \ll a)$$



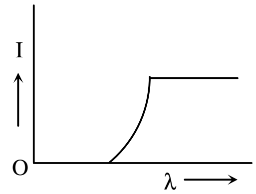
66. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is:

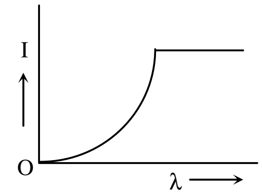
(1) $I_0/2$ (2) $I_0/4$

(3) $I_0/8$ (4) I_0

Sol. **(2)**

67. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows:

(1)


(2)


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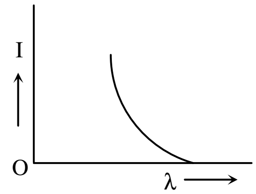

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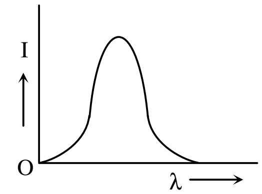
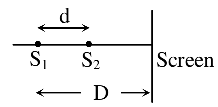


Sol. **(3)**

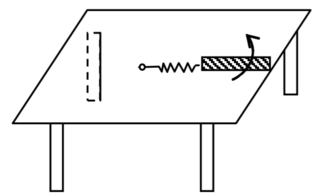
68. Two coherent point sources S_1 and S_2 are separated by a small distance 'd' as shown. The fringes obtained on the screen will be:



(1) straight lines (2) semi-circles
(3) concentric circles (4) points

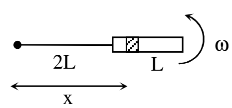
Sol. **(3)**

69. A metallic rod of length 'ℓ' is tied to a string of length 2ℓ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field 'B' in the region, the e.m.f. induced across the ends of the rod is:



(1) $\frac{3B\omega\ell^2}{2}$ (2) $\frac{4B\omega\ell^2}{2}$
(3) $\frac{5B\omega\ell^2}{2}$ (4) $\frac{2B\omega\ell^2}{2}$

Sol. **(3)**



$de = B(\omega x)\cdot dx$

$e = B\omega\int_{2L}^{3L} x\,dx$

$= \frac{5B\omega L^2}{2}$

70. In a hydrogen like atom electron makes transition from an energy level with quantum number n to another with quantum number (n – 1). If n >> 1, the frequency of radiation emitted is proportional to

(1) $\frac{1}{n^2}$ (2) $\frac{1}{n^{3/2}}$
(3) $\frac{1}{n^3}$ (4) $\frac{1}{n}$

Sol. **(3)**

$\nu \propto \left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$

$\propto \frac{(2n-1)}{n^2(n-1)^2}$

$\propto \frac{1}{n^3}$ (since $n \gg 1$)

71. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, density of liquid is ρ and L is its latent heat of vaporization.

(1) $\sqrt{T/\rho L}$ (2) $T/\rho L$

(3) $2T/\rho L$ (4) $\rho L/T$

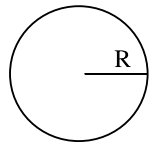
Sol. **(3)**

$\rho 4\pi R^2 \Delta R L = T\, 4\pi[R^2 - (R - \Delta R)^2]$

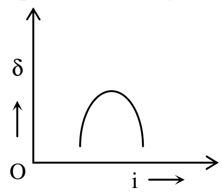
$\rho R^2 \Delta R L = T[R^2 - R^2 + 2R\Delta R - \Delta R^2]$

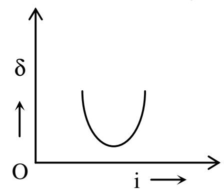
$\rho R^2 \Delta R L = T 2R\Delta R$ (ΔR is very small)

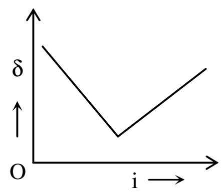
$R = \dfrac{2T}{\rho L}$.

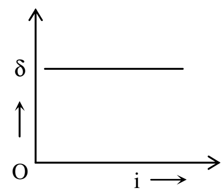


72. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by:

(1)


(2)


(3)


(4)


Sol. **(2)**

73. Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then:

(1) $[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$ (2) $[\varepsilon_0] = [M^{-1}L^2T^{-1}A^{-2}]$

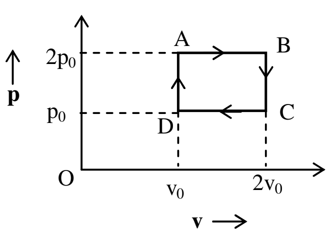
(3) $[\varepsilon_0] = [M^{-1}L^2T^{-1}A]$ (4) $[\varepsilon_0] = [M^{-1}L^{-3}T^2A]$

Sol. **(1)**

73. $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{r^2} = F$

$\varepsilon_0 = \dfrac{[A^2T^2]}{[MLT^{-2}L^2]} = [M^{-1}L^{-3}A^2T^4]$

74.



The above p-v diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat extracted from the source in a single cycle is

(1) $\left(\dfrac{13}{2}\right)p_0v_0$ (2) $\left(\dfrac{11}{2}\right)p_0v_0$

(3) $4p_0v_0$ (4) p_0v_0

Sol. **(2)**

Heat is extracted from the source in path DA and AB is

$$\Delta Q = \frac{3}{2}R\left(\frac{P_0V_0}{R}\right) + \frac{5}{2}R\left(\frac{2P_0V_0}{R}\right) = \frac{13}{2}P_0V_0$$

75. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1 %. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11}N/m^2 respectively?

(1) 178.2 Hz
(2) 200.5 Hz
(3) 770 Hz
(4) 188.5 Hz

Sol. **(1)**

Fundamental frequency $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

$= \frac{1}{2\ell}\sqrt{\frac{T}{A\rho}}$

$= \frac{1}{2\ell}\sqrt{\frac{\text{stress}}{\rho}} = \frac{1}{2\times1.5}\sqrt{\frac{2.2\times10^{11}\times10^{-2}}{7.7\times10^3}}$.

76. This question has statement I and statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement- I: Higher the range, greater is the resistance of ammeter.

Statement- II: To increase the range of ammeter, additional shunt needs to be used across it.

(1) Statement – I is true, Statement – II is true, Statement – II is not the correct explanation of Statement–I.
(2) Statement – I is true, statement – II is false.
(3) Statement – I is false, Statement – II is true
(4) Statement – I is true, Statement – II is true, Statement – II is the correct explanation of statement- I .

Sol. **(3)**

For Ammeter, $S = \frac{I_gG}{I - I_g}$

So for I to increase, S should decrease, so additional S can be connected across it.

77. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R?

(1) $\frac{2GmM}{3R}$
(2) $\frac{GmM}{2R}$
(3) $\frac{GmM}{3R}$
(4) $\frac{5GmM}{6R}$

Sol. **(4)**

$T.\,E_f = -\frac{GMm}{6R}$

$T.\,E_i = -\frac{GMm}{R}$

$\Delta W = T.E_f - T.E_i = \frac{5GMm}{6R}$

78. A projectile is given an initial velocity of $\left(\hat{i} + 2\hat{j}\right)$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s^2, the equation of its trajectory is:

(1) $y = 2x - 5x^2$
(2) $4y = 2x - 5x^2$
(3) $4y = 2x - 25x^2$
(4) $y = x - 5x^2$

Sol. **(1)**

$x = t$

$y = 2t - 5t^2$

Equation of trajectory is $y = 2x - 5x^2$

79. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then :

(1) $3C_1 = 5C_2$ (2) $3C_1 + 5C_2 = 0$

(3) $9C_1 = 4C_2$ (4) $5C_1 = 3C_2$

Sol. **(1)**

$120C_1 = 200C_2$

$6C_1 = 10C_2$

$3C_1 = 5C_2$

80. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip?

(1) $\frac{r\omega_0}{3}$ (2) $\frac{r\omega_0}{2}$

(3) $r\omega_0$ (4) $\frac{r\omega_0}{4}$

Sol. **(2)**

From conservation of angular momentum about any fix point on the surface

$mr^2\omega_0 = 2mr^2\omega$

$\therefore \quad \omega = \frac{\omega_0}{2}$

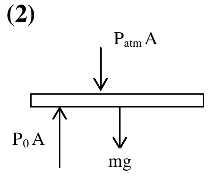
$\therefore \quad V_{CM} = \frac{\omega_0 r}{2}$

81. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency:

(1) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$ (2) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$

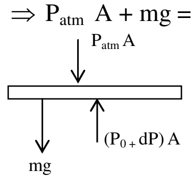
(3) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$ (4) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$

Sol. **(2)**



FBD of piston at equilibrium

$\Rightarrow$ P_{atm} A + mg = P_0 A ...(1)



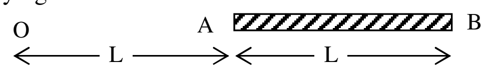
FBD of piston when piston is pushed down a distance x

$P_{atm} + mg - (P_0 + dP)A = m\frac{d^2x}{dt^2}$ ...(2)

Process is adiabatic $\Rightarrow PV^{\gamma} = C \Rightarrow -dP = \frac{\gamma P dV}{V}$

Using 1, 2, 3 me get $f = \frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$

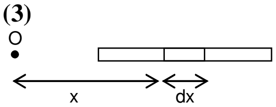
82. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at a distance L from the end A is:



(1) $\frac{3Q}{4\pi\varepsilon_0 L}$ (2) $\frac{Q}{4\pi\varepsilon_0 L \ln 2}$

(3) $\frac{Q\ln 2}{4\pi\varepsilon_0 L}$ (4) $\frac{Q}{8\pi\varepsilon_0 L}$

Sol. **(3)**



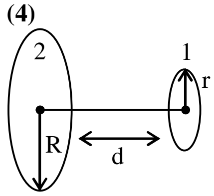
$V = \int_{x=L}^{x=2L} \frac{k}{x}\left(\frac{Q}{L}\right)dx = \frac{Q\ln 2}{4\pi\varepsilon_0 L}$

83. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is

(1) 6×10^{-11} weber (2) 3.3×10^{-11} weber

(3) 6.6×10^{-9} weber (4) 9.1×10^{-11} weber

Sol. **(4)**



Let M_{12} be the coefficient of mutual induction between loops

$\phi_1 = M_{12} i_2$

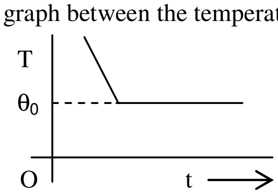
$\Rightarrow \frac{\mu_0 i_2 R^2}{2(d^2 + R^2)^{3/2}}\pi r^2 = M_{12} i_2$

$\Rightarrow M_{12} = \frac{\mu_0 R^2 \pi r^2}{2(d^2 + R^2)^{3/2}}$

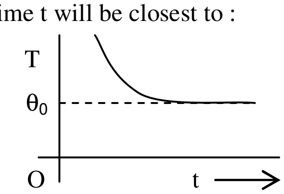
$\phi_2 = M_{12} i_1 \Rightarrow \phi_2 = 9.1 \times 10^{-11}$ weber

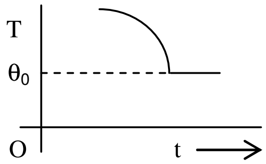
84. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0 the graph between the temperature T of the metal and time t will be closest to :

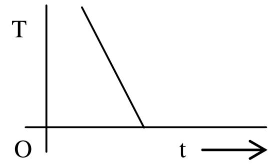
(1)



(2)



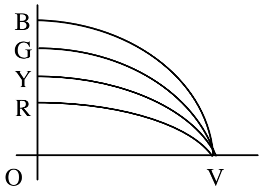
(3) 

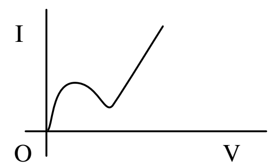
(4) 

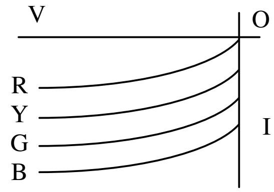
Sol. **(2)**

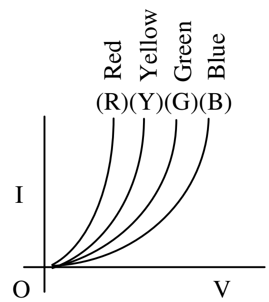
The temperature goes on decreasing with time (non-linearly) The rate of decrease will be more initially which is depicted in the second graph.

85. The I – V characteristic of an LED is

(1) 

(2) 

(3) 

(4) 

Sol. **(4)**

For LED, in forward bias, intensity increases with voltage.

86. This question has Statement I and Statement II. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement – I : A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $f=\left(\frac{m}{M+m}\right)$.

Statement – II : Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(1) Statement – I is true, Statement – II is true, Statement – II is not a correct explanation of Statement – I.
(2) Statement – I is true, Statement – II is false.
(3) Statement – I is false, Statement – II is true
(4) Statement – I is true, Statement – II is true, Statement – II is a correct explanation of Statement – I.

Sol. **(3)**

Loss of energy is maximum when collision is inelastic as in an inelastic collision there will be maximum deformation.

KE in COM frame is $\frac{1}{2}\left(\frac{Mm}{M+m}\right)V_{rel}^2$

$KE_i = \frac{1}{2}\left(\frac{Mm}{M+m}\right)V^2 \qquad KE_f = 0 \ (\because V_{rel} = 0)$

Hence loss in energy is $\frac{1}{2}\left(\frac{Mm}{M+m}\right)V^2$

$\Rightarrow f = \frac{M}{M+m}$

87. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude is 5s. In another 10s it will decrease to α times its original magnitude, where α equals.

(1) 0.81 (2) 0.729
(3) 0.6 (4) 0.7

Sol. **(2)**

$A = A_0 e^{-kt}$

$\Rightarrow\ 0.9A_0 = A_0 e^{-5k}$

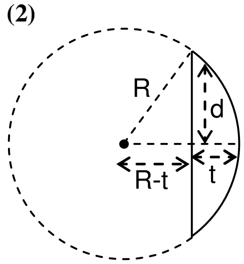
and $\alpha A_0 = A_0 e^{-15k}$

solving $\Rightarrow\ \alpha = 0.729$

88. Diameter of plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is :

(1) 20 cm (2) 30 cm
(3) 10 cm (4) 15 cm

Sol. **(2)**



$R^2 = d^2 + (R-t)^2$

$R^2 - d^2 = R^2 \left\{1 - \frac{t}{R}\right\}^2$

$1 - \frac{d^2}{R^2} = 1 - \frac{2t}{R}$

$R = \frac{(3)^2}{2\times(0\cdot 3)} = \frac{90}{6} = 15\,\text{cm}$

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{15}\right)$

$f = 30\,\text{cm}$

89. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is :

(1) 6 V/m (2) 9 V/m
(3) 12 V/m (4) 3 V/m

Sol. **(1)**

$E_0 = CB_0$

$= 3 \times 10^8 \times 20 \times 10^{-9}$

= 6 V/m

90. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2 respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid - point O of the line joining their centres is close to (Horizontal component of earth's magnetic induction is 3.6×10^{-5} Wb/m^2)

(1) 2.56×10^{-4} Wb/m^2

(2) 3.50×10^{-4} Wb/m^2

(3) 5.80×10^{-4} Wb/m^2

(4) 3.6×10^{-5} Wb/m^2

Sol. **(1)**

$$B_{net} = B_{M_1} + B_{M_2} + B_H$$

$$= \frac{\mu_0 M_1}{4\pi x^3} + \frac{\mu_0 M_2}{4\pi x^3} + B_H$$

$$= \frac{\mu_0}{4\pi x^3}\left(M_1 + M_2\right) + B_H$$

$$= \frac{10^{-7}}{10^{-3}} \times 2.2 + 3.6 \times 10^{-5}$$

$$= 2.56 \times 10^{-4} \text{ Wb/m}^2$$