PART - III: MATHEMATICS

SECTION – 1 : (One or more option correct Type)

This section contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

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41. For $a \in \mathbb{R}$ (the set of all real numbers), $a \neq -1$, $\lim_{n\to\infty} \frac{(1^a + 2^a + ... + n^a)}{(n+1)^{a-1}[(na+1)+(na+2)+...+(na+n)]} = \frac{1}{60}$.

Then a =

(A) 5 (B) 7

(C) $\frac{-15}{2}$ (D) $\frac{-17}{2}$

Sol. **(B, D)**

$$\text{Required limit} = \frac{\int_0^1 x^a dx}{\int_0^1 (a+x)\,dx} = \frac{2}{(2a+1)(a+1)} = \frac{2}{120}$$

$\Rightarrow a = 7$ or $-\frac{17}{2}$.

*42. Circle(s) touching x-axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ on y-axis is (are)

(A) $x^2 + y^2 - 6x + 8y + 9 = 0$ (B) $x^2 + y^2 - 6x + 7y + 9 = 0$

(C) $x^2 + y^2 - 6x - 8y + 9 = 0$ (D) $x^2 + y^2 - 6x - 7y + 9 = 0$

Sol. **(A), (C)**

Equation of circle can be written as

$(x - 3)^2 + y^2 + \lambda (y) = 0$

$\Rightarrow x^2 + y^2 - 6x + \lambda y + 9 = 0.$

Now, $(\text{radius})^2 = 7 + 9 = 16$

$\Rightarrow 9 + \frac{\lambda^2}{4} - 9 = 16$

$\Rightarrow \lambda^2 = 64 \Rightarrow \lambda = \pm 8.$

$\therefore$ Equation is $x^2 + y^2 - 6x \pm 8y + 9 = 0.$

43. Two lines $L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$ and $L_2 : x = \alpha, \frac{y}{-1} = \frac{z}{2-\alpha}$ are coplanar. Then α can take value(s)

(A) 1 (B) 2

(C) 3 (D) 4

Sol. **(A, D)**

$$\frac{x-5}{0} = \frac{y-0}{3-\alpha} = \frac{z-0}{-2}$$

$\frac{x-\alpha}{0}=\frac{y}{-1}=\frac{z}{2-\alpha}$

will be coplanar if shortest distance is zero

$\Rightarrow \begin{vmatrix} 5-\alpha & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$

$(5-\alpha)(\alpha^2-5\alpha+4)=0$, $\alpha = 1, 4, 5$

so $\alpha = 1, 4$

Alternate Solution:

As $x = 5$ and $x = \alpha$ are parallel planes so the remaining two planes must be coplanar.

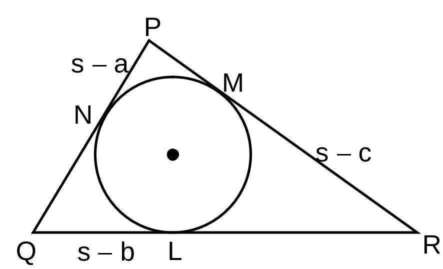
So, $\frac{3-\alpha}{-1}=\frac{-2}{2-\alpha} \Rightarrow \alpha^2-5\alpha+4=0 \Rightarrow \alpha = 1, 4.$

*44. In a triangle PQR, P is the largest angle and $\cos P = \frac{1}{3}$. Further the incircle of the triangle touches the sides PQ, QR and RP at N, L and M respectively, such that the lengths of PN, QL and RM are consecutive even integers. Then possible length(s) of the side(s) of the triangle is (are)

(A) 16 (B) 18

(C) 24 (D) 22

Sol. **(B), (D)**



Let

$s-a = 2k-2$, $s-b = 2k$, $s-c = 2k+2$, $k \in I$, $k > 1$

Adding we get,

$s = 6k$

So, $a = 4k+2$, $b = 4k$, $c = 4k-2$

Now, $\cos P = \frac{1}{3}$

$\Rightarrow \frac{b^2+c^2-a^2}{2bc}=\frac{1}{3} \Rightarrow 3[(4k)^2+(4k-2)^2-(4k+2)^2] = 2 \times 4k(4k-2)$

$\Rightarrow 3[16k^2 - 4(4k)\times 2] = 8k(4k-2)$

$\Rightarrow 48k^2 - 96k = 32k^2 - 16k$

$\Rightarrow 16k^2 = 80k \Rightarrow k = 5$

So, sides are 22, 20, 18

*45. Let $w = \frac{\sqrt{3}+i}{2}$ and $P = \{w^n : n = 1, 2, 3, \ldots\}$. Further $H_1 = \left\{z \in C : \operatorname{Re} z > \frac{1}{2}\right\}$ and $H_2 = \left\{z \in C : \operatorname{Re} z < \frac{-1}{2}\right\}$, where C is the set of all complex numbers. If $z_1 \in P \cap H_1$, $z_2 \in P \cap H_2$ and O represents the origin, then $\angle z_1 O z_2 =$

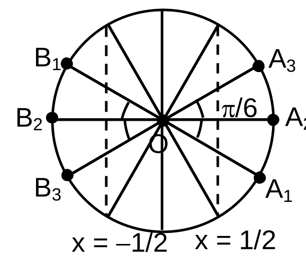
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$

(C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

Sol. **(C), (D)**

$w = \dfrac{\sqrt{3}+i}{2} = e^{\frac{i\pi}{6}}$, so $w^n = e^{i\left(\frac{n\pi}{6}\right)}$

Now, for z_1, $\cos\dfrac{n\pi}{6} > \dfrac{1}{2}$ and for z_2, $\cos\dfrac{n\pi}{6} < -\dfrac{1}{2}$



Possible position of z_1 are A_1, A_2, A_3 whereas of z_2 are B_1, B_2, B_3 (as shown in the figure)

So, possible value of $\angle z_1Oz_2$ according to the given options is $\dfrac{2\pi}{3}$ or $\dfrac{5\pi}{6}$.

*46. If $3^x = 4^{x-1}$, then x =

(A) $\dfrac{2\log_3 2}{2\log_3 2-1}$ (B) $\dfrac{2}{2-\log_2 3}$

(C) $\dfrac{1}{1-\log_4 3}$ (D) $\dfrac{2\log_2 3}{2\log_2 3-1}$

Sol. **(A, B, C)**

$\log_2 3^x = (x-1)\log_2 4 = 2(x-1)$

$\Rightarrow x\log_2 3 = 2x - 2$

$\Rightarrow x = \dfrac{2}{2-\log_2 3}$

Rearranging, we get

$x = \dfrac{2}{2-\dfrac{1}{\log_3 2}} = \dfrac{2\log_3 2}{2\log_3 2-1}$

Rearranging again,

$x = \dfrac{\log_3 4}{\log_3 4-1} = \dfrac{\dfrac{1}{\log_4 3}}{\dfrac{1}{\log_4 3}-1} = \dfrac{1}{1-\log_4 3}$.

47. Let ω be a complex cube root of unity with ω ≠ 1 and $P = [p_{ij}]$ be a n × n matrix with $p_{ij} = \omega^{i+j}$. Then $P^2 \neq 0$, when n =

(A) 57 (B) 55

(C) 58 (D) 56

Sol. **(B, C, D)**

$$P = \begin{bmatrix} \omega^2 & \omega^3 & \omega^4 & ..... & \omega^{n+2} \\ \omega^3 & \omega^4 & \omega^5 & ..... & \omega^{n+3} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \omega^{n+2} & \omega^{n+3} & ..... & ..... & \omega^{2n+4} \end{bmatrix}$$

$$P^2 = \begin{bmatrix} \omega^4+\omega^6..... & \omega^5+\omega^7+\omega^9 & ..... & ..... \\ \omega^5+\omega^7+\omega^9..... & ..... & ..... & ..... \\ \vdots & \vdots & \vdots & ..... \\ \omega^{n+4}+\omega^{n+6}..... & ..... & ..... & \omega^{2n+4}+\omega^{2n+6}..... \end{bmatrix}$$

P^2 = Null matrix if n is a multiple of 3

48. The function $f(x) = 2|x| + |x+2| - ||x+2| - 2|x||$ has a local minimum or a local maximum at x =

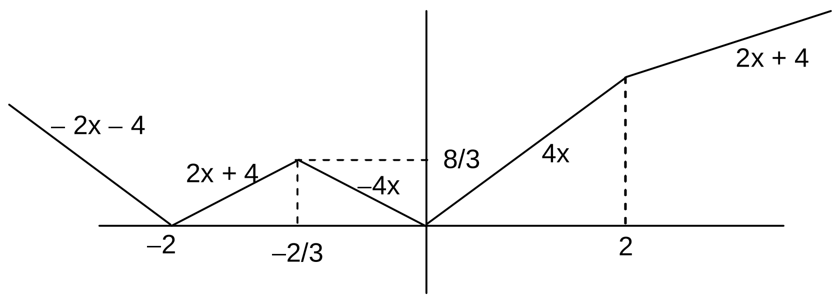
(A) -2 (B) $\frac{-2}{3}$

(C) 2 (D) $\frac{2}{3}$

Sol. **(A), (B)**

As, $\frac{f(x)+g(x)-|f(x)-g(x)|}{2}$ = Min (f (x), g (x))

$\Rightarrow \frac{2|x|+|x+2|-||x+2|-2|x||}{2}$ = Min (|2x|, |x + 2|)



According to the figure shown, points of local minima/maxima are x = $-2, \frac{-2}{3}, 0$.

SECTION – 2 : (Paragraph Type)

This section contains 6 multiple choice questions relating to three paragraphs with two questions on each paragraph. Each question has four choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

Paragraph for Questions 49 and 50

Let $f : [0, 1] \to \mathbb{R}$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable, f(0) = f(1) = 0 and satisfies $f''(x) - 2f'(x) + f(x) \ge e^x$, $x \in [0, 1]$.

49. Which of the following is true for $0 < x < 1$?

(A) $0 < f(x) < \infty$ (B) $-\frac{1}{2} < f(x) < \frac{1}{2}$

(C) $-\frac{1}{4} < f(x) < 1$ (D) $-\infty < f(x) < 0$

Sol. **(D)**

Let $g(x) = e^{-x} f(x)$

and $g''(x) > 1 > 0$

So, g (x) is concave upward and g (0) = g (1) = 0

Hence, $g(x) < 0 \ \forall \ x \in (0, 1)$

$\Rightarrow e^{-x} f(x) < 0$

$f(x) < 0 \ \forall \ x \in (0, 1)$

Alternate Solution

$f''(x) - 2f'(x) + f(x) \ge e^x$

$$\Rightarrow \left(f(x)e^{-x} - \frac{x^2}{2}\right)'' \ge 0$$

Let $g(x) = f(x)e^{-x} - \frac{x^2}{2}$

$g(0) = 0,\ g(1) = -\frac{1}{2}$

Since g is concave up so it will always lie below the chord joining the extremities which is $y = -\frac{x}{2}$

$$\Rightarrow f(x)e^{-x} - \frac{x^2}{2} < -\frac{x}{2}$$

$$\Rightarrow f(x) < \frac{(x^2 - x)e^x}{2} < 0\ \ \forall x \in (0, 1)$$

50. If the function $e^{-x} f(x)$ assumes its minimum in the interval [0, 1] at $x = \frac{1}{4}$, which of the following is true ?

(A) $f'(x) < f(x), \frac{1}{4} < x < \frac{3}{4}$

(B) $f'(x) > f(x), 0 < x < \frac{1}{4}$

(C) $f'(x) < f(x), 0 < x < \frac{1}{4}$

(D) $f'(x) < f(x), \frac{3}{4} < x < 1$

Sol. **C**

Let, $g(x) = e^{-x} f(x)$

As $g''(x) > 0$ so $g'(x)$ is increasing.

So, for $x < 1/4$, $g'(x) < g'(1/4) = 0$

$\Rightarrow (f'(x) - f(x))e^{-x} < 0$

$\Rightarrow f'(x) < f(x)$ in (0, 1/4).

Paragraph for Questions 51 and 52

Let PQ be a focal chord of the parabola $y^2 = 4ax$. The tangents to the parabola at P and Q meet at a point lying on the line $y = 2x + a$, $a > 0$.

*51. Length of chord PQ is

(A) 7a

(B) 5a

(C) 2a

(D) 3a

Sol. **(B)**

Let $P(at^2, 2at)$, $Q\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$ as PQ is focal chord

Point of intersection of tangents at P and Q

$\left(-a,\ a\left(t - \frac{1}{t}\right)\right)$

as point of intersection lies on $y = 2x + a$

$$\Rightarrow a\left(t - \frac{1}{t}\right) = -2a + a$$

$$t - \frac{1}{t} = -1 \Rightarrow \left(t + \frac{1}{t}\right)^2 = 5$$

length of focal chord = $a\left(t + \frac{1}{t}\right)^2 = 5a$

*52. If chord PQ subtends an angle θ at the vertex of $y^2 = 4ax$, then $\tan\theta =$

(A) $\frac{2}{3}\sqrt{7}$ (B) $\frac{-2}{3}\sqrt{7}$

(C) $\frac{2}{3}\sqrt{5}$ (D) $\frac{-2}{3}\sqrt{5}$

Sol. **(D)**

Angle made by chord PQ at vertex (0, 0) is given by

$$\tan\theta = \left(\frac{\frac{2}{t}+2t}{1-4}\right) = \frac{2\left(\frac{1}{t}+t\right)}{-3} = \frac{-2}{3}\sqrt{5}$$

Paragraph for Questions 53 and 54

Let $S = S_1 \cap S_2 \cap S_3$, where

$S_1 = \{z \in C : |z| < 4\}$, $S_2 = \left\{z \in C : \text{Im}\left[\frac{z-1+\sqrt{3}i}{1-\sqrt{3}i}\right] > 0\right\}$ and $S_3 = \{z \in C : \text{Re } Z > 0\}$.

*53. Area of S =

(A) $\frac{10\pi}{3}$ (B) $\frac{20\pi}{3}$

(C) $\frac{16\pi}{3}$ (D) $\frac{32\pi}{3}$

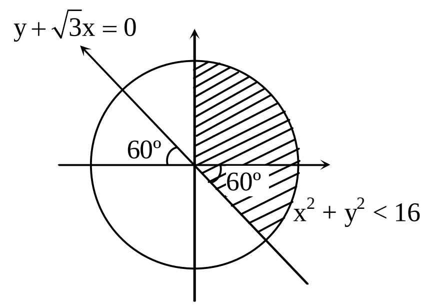
Sol. **(B)**

Area of region $S_1 \cap S_2 \cap S_3$ = shaded area

$$= \frac{\pi \times 4^2}{4} + \frac{4^2 \times \pi}{6}$$

$$= 4^2\pi\left\{\frac{1}{4}+\frac{1}{6}\right\}$$

$$= \frac{20\pi}{3}$$



*54. $\min_{z\in S}|1-3i-z| =$

(A) $\frac{2-\sqrt{3}}{2}$ (B) $\frac{2+\sqrt{3}}{2}$

(C) $\frac{3-\sqrt{3}}{2}$ (D) $\frac{3+\sqrt{3}}{2}$

Sol. **(C)**

Distance of (1, –3) from $y+\sqrt{3}x = 0$

$$> \left|\frac{-3+\sqrt{3}\times 1}{2}\right|$$

$$> \frac{3-\sqrt{3}}{2}$$

Paragraph for Questions 55 and 56

A box B_1 contains 1 white ball, 3 red balls and 2 black balls. Another box B_2 contains 2 white balls, 3 red balls and 4 black balls. A third box B_3 contains 3 white balls, 4 red balls and 5 black balls.

55. If 1 ball is drawn from each of the boxes B_1, B_2 and B_3, the probability that all 3 drawn balls are of the same colour is

(A) $\frac{82}{648}$ (B) $\frac{90}{648}$

(C) $\frac{558}{648}$ (D) $\frac{566}{648}$

Sol. **(A)**

P (required) = P (all are white) + P (all are red) + P (all are black)

$= \frac{1}{6}\times\frac{2}{9}\times\frac{3}{12}+\frac{3}{6}\times\frac{3}{9}\times\frac{4}{12}+\frac{2}{6}\times\frac{4}{9}\times\frac{5}{12}$

$= \frac{6}{648}+\frac{36}{648}+\frac{40}{648}=\frac{82}{648}$.

56. If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red, the probability that these 2 balls are drawn from box B_2 is

(A) $\frac{116}{181}$ (B) $\frac{126}{181}$

(C) $\frac{65}{181}$ (D) $\frac{55}{181}$

Sol. **(D)**

Let A : one ball is white and other is red

E_1 : both balls are from box B_1

E_2 : both balls are from box B_2

E_3 : both balls are from box B_3

Here, P (required) = $P\left(\frac{E_2}{A}\right)$

$$= \frac{P\left(\frac{A}{E_2}\right)\cdot P(E_2)}{P\left(\frac{A}{E_1}\right)\cdot P(E_1)+P\left(\frac{A}{E_2}\right)\cdot P(E_2)+P\left(\frac{A}{E_3}\right)\cdot P(E_3)}$$

$$= \frac{\frac{{}^2C_1\times{}^3C_1}{{}^9C_2}\times\frac{1}{3}}{\frac{{}^1C_1\times{}^3C_1}{{}^6C_2}\times\frac{1}{3}+\frac{{}^2C_1\times{}^3C_1}{{}^9C_2}\times\frac{1}{3}+\frac{{}^3C_1\times{}^4C_1}{{}^{12}C_2}\times\frac{1}{3}} = \frac{\frac{1}{6}}{\frac{1}{5}+\frac{1}{6}+\frac{2}{11}} = \frac{55}{181}.$$

SECTION – 3 : (Matching list Type)

This section contains 4 multiple choice questions. Each question has matching lists. The codes for the lists have choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

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*57. Match List I with List II and select the correct answer using the code given below the lists :

	List – I		List – II
P.	$\left(\frac{1}{y^2}\left(\frac{\cos\left(\tan^{-1}y\right)+y\sin\left(\tan^{-1}y\right)}{\cot\left(\sin^{-1}y\right)+\tan\left(\sin^{-1}y\right)}\right)+y^4\right)^{1/2}$ takes value	1.	$\frac{1}{2}\sqrt{\frac{5}{3}}$
Q.	If cosx + cosy + cosz = 0 = sinx + siny + sinz then possible value of $\cos\frac{x-y}{2}$ is	2.	$\sqrt{2}$
R.	If $\cos\left(\frac{\pi}{4}-x\right)$cos2x + sinx sin2x secx = cosx sin2x secx + $\cos\left(\frac{\pi}{4}+x\right)$cos2x then possible value of secx is	3.	$\frac{1}{2}$
S.	If $\cot\left(\sin^{-1}\sqrt{1-x^2}\right)=\sin\left(\tan^{-1}\left(x\sqrt{6}\right)\right)$, x ≠ 0, then possible value of x is	4.	1

Codes :

	P	Q	R	S
(A)	4	3	1	2
(B)	4	3	2	1
(C)	3	4	2	1
(D)	3	4	1	2

Sol. **(B)**

$$P \rightarrow \frac{\cos\left(\tan^{-1}y\right)+y\sin\left(\tan^{-1}y\right)}{\cot\left(\sin^{-1}y\right)+\tan\left(\sin^{-1}y\right)}$$

$$= \frac{\frac{1}{\sqrt{1+y^2}}+\frac{y^2}{\sqrt{1+y^2}}}{\frac{\sqrt{1-y^2}}{y}+\frac{y}{\sqrt{1-y^2}}} = \frac{\sqrt{1+y^2}}{\frac{1}{y\sqrt{1-y^2}}} = y\sqrt{1-y^4}$$

$$\Rightarrow \frac{1}{y^2}\left(\frac{\cos\left(\tan^{-1}y\right)+y\sin\left(\tan^{-1}y\right)}{\cot\left(\sin^{-1}y\right)+\tan\left(\sin^{-1}y\right)}\right)^2+y^4$$

$$= \frac{1}{y^2}\left(y^2\left(1-y^4\right)\right)+y^4 = 1-y^4+y^4 = 1$$

$Q \rightarrow \cos x + \cos y + \cos z = 0$

$\sin x + \sin y + \sin z = 0$

$\cos x + \cos y = -\cos z$ ..... (1)

$\sin x + \sin y = -\sin z$ ..... (2)

$(1)^2 + (2)^2$

$1 + 1 + 2(\cos x \cos y + \sin x \sin y) = 1$

$2 + 2\cos(x - y) = 1$

$2\cos(x - y) = -1$

$\cos(x - y) = -\frac{1}{2}$

$2\cos^2\left(\frac{x-y}{2}\right)-1=-\frac{1}{2}$

$2\cos^2\left(\frac{x-y}{2}\right)=\frac{1}{2}$

$$\cos^2\left(\frac{x-y}{2}\right)=\frac{1}{4}$$

$$\cos\left(\frac{x-y}{2}\right)=\frac{1}{2}$$

$R \to \cos\left(\frac{\pi}{4}-x\right)\cos 2x+\sin x\sin 2x\sec x$

$$= \cos x\sin 2x\sec x+\cos\left(\frac{\pi}{4}+x\right)\cos 2x$$

$$\left[\cos\left(\frac{\pi}{4}-x\right)-\cos\left(\frac{\pi}{4}+x\right)\right]\cos 2x = (\cos x \sin 2x - \sin x \sin 2x) \sec x$$

$$\frac{2}{\sqrt{2}}\sin x\cos 2x=(\cos x-\sin x)\sin 2x\sec x$$

$$\sqrt{2}\sin x\cos 2x=(\cos x-\sin x)2\sin x$$

$$\frac{1}{\sqrt{2}}=\frac{1}{\cos x+\sin x} \Rightarrow x=\frac{\pi}{4}$$

$$\sec x=\sec\frac{\pi}{4}=\sqrt{2}$$

$S \to \cot\left(\sin^{-1}\sqrt{1-x^2}\right)$

$$\cot\alpha=\frac{x}{\sqrt{1-x^2}}$$

$$\tan^{-1}\left(x\sqrt{6}\right)=\phi$$

$$\sin\phi=\frac{x\sqrt{6}}{\sqrt{6x^2+1}}$$

$$\Rightarrow \frac{x}{\sqrt{1-x^2}}=\frac{x\sqrt{6}}{\sqrt{6x^2+1}}$$

$$6x^2 + 1 = 6 - 6x^2$$

$$12x^2 = 5$$

$$x=\sqrt{\frac{5}{12}}=\frac{1}{2}\sqrt{\frac{5}{3}}$$

*58. A line $L : y = mx + 3$ meets y-axis at $E(0, 3)$ and the arc of the parabola $y^2 = 16x$, $0 \le y \le 6$ at the point $F(x_0, y_0)$. The tangent to the parabola at $F(x_0, y_0)$ intersects the y-axis at $G(0, y_1)$. The slope m of the line L is chosen such that the area of the triangle EFG has a local maximum.

Match List I with List II and select the correct answer using the code given below the lists :

List – I		List – II	
P.	m =	1.	$\frac{1}{2}$
Q.	Maximum area of ΔEFG is	2.	4
R.	y_0 =	3.	2
S.	y_1 =	4.	1

Codes :

	P	Q	R	S
(A)	4	1	2	3
(B)	3	4	1	2
(C)	1	3	2	4
(D)	1	3	4	2

Sol. **(A)**

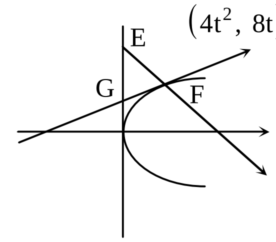
$A(t) = 2t^2(3 - 4t)$

For max. A(t), $t = \frac{1}{2}$

$\Rightarrow m = 1$

$\Rightarrow A(t)\Big|_{max.} = \frac{1}{2}$ sq. units

$y_0 = 4$ and $y_1 = 2$



59. Match List I with List II and select the correct answer using the code given below the lists :

	List – I		List – II
P.	Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 2. Then the volume of the parallelepiped determined by vectors $2(\vec{a}\times\vec{b}), 3(\vec{b}\times\vec{c})$ and $(\vec{c}\times\vec{a})$ is	1.	100
Q.	Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 5. Then the volume of the parallelepiped determined by vectors $3(\vec{a}+\vec{b}), (\vec{b}+\vec{c})$ and $2(\vec{c}+\vec{a})$ is	2.	30
R.	Area of a triangle with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 20. Then the area of the triangle with adjacent sides determined by vectors $(2\vec{a}+3\vec{b})$ and $(\vec{a}-\vec{b})$ is	3.	24
S.	Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then the area of the parallelogram with adjacent sides determined by vectors $(\vec{a}+\vec{b})$ and $\vec{a}$ is	4.	60

Codes :

	P	Q	R	S
(A)	4	2	3	1
(B)	2	3	1	4
(C)	3	4	1	2
(D)	1	4	3	2

Sol. **(C)**

$P \rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right] = 2$

$$\left[2\vec{a}\times\vec{b}\ \ 3\vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right] = 6\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2 = 6\times 4 = 24$$

$Q \rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right] = 5$

$$6\left[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}\right] = 12\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 60$$

$R \rightarrow \frac{1}{2}\left|\vec{a}\times\vec{b}\right| = 20$

$$\frac{1}{2}\left|(2\vec{a}+3\vec{b})\times(\vec{a}-\vec{b})\right|$$

$\frac{1}{2}\left|-2(\vec{a}\times\vec{b})-3(\vec{a}\times\vec{b})\right|$

$\frac{5}{2}\times 40 = 100$

S → $|\vec{a}\times\vec{b}| = 30$

$\Rightarrow \left|(\vec{a}+\vec{b})\times\vec{a}\right| = |\vec{b}\times\vec{a}| = 30$

60. Consider the lines $L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}$, $L_2 : \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and the planes $P_1 : 7x + y + 2z = 3$, $P_2 : 3x + 5y - 6z = 4$. Let ax + by + cz = d be the equation of the plane passing through the point of intersection of lines L_1 and L_2, and perpendicular to planes P_1 and P_2.
Match List I with List II and select the correct answer using the code given below the lists :

List – I		List – II	
P.	a =	1.	13
Q.	b =	2.	–3
R.	c =	3.	1
S.	d =	4.	–2

Codes :

	P	Q	R	S
(A)	3	2	4	1
(B)	1	3	4	2
(C)	3	2	1	4
(D)	2	4	1	3

Sol. **(A)**

Plane perpendicular to P_1 and P_2 has Direction Ratios of normal

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix} = -16\hat{i} + 48\hat{j} + 32\hat{k} \quad \dots (1)$$

For point of intersection of lines

$(2\lambda_1 + 1, -\lambda_1, \lambda_1 - 3) \equiv (\lambda_2 + 4, \lambda_2 - 3, 2\lambda_2 - 3)$

$\Rightarrow 2\lambda_1 + 1 = \lambda_2 + 4$ or $2\lambda_1 - \lambda_2 = 3$

$-\lambda_1 = \lambda_2 - 3$ or $\lambda_1 + \lambda_2 = 3$

$\Rightarrow \lambda_1 = 2, \lambda_2 = 1$

$\therefore$ Point is (5, – 2, – 1) … (2)

From (1) and (2), required plane is

$-1(x-5) + 3(y+2) + 2(z+1) = 0$

or $-x + 3y + 2z = -13$

$x - 3y - 2z = 13$

$\Rightarrow a = 1, b = -3, c = -2, d = 13.$