# IITJEE 2013

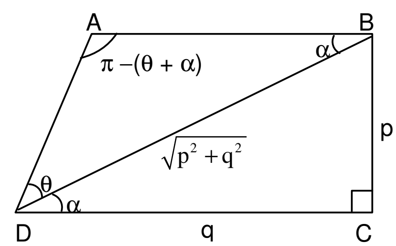
## PART B – MATHEMATICS

31. The circle passing through (1, –2) and touching the axis of x at (3, 0) also passes through the point
(1) (2, –5)
(2) (5, –2)
(3) (–2, 5)
(4) (–5, 2)

*Sol.* **(2)**
$(x - 3)^2 + y^2 + \lambda y = 0$
The circle passes through (1, – 2)
$\Rightarrow 4 + 4 - 2\lambda = 0 \Rightarrow \lambda = 4$
$(x - 3)^2 + y^2 + 4y = 0 \Rightarrow$ Clearly (5, – 2) satisfies.

32. ABCD is a trapezium such that AB and CD are parallel and BC⊥CD. If ∠ADB = θ, BC = p and CD = q, then AB is equal to
(1) $\dfrac{p^2 + q^2 \cos\theta}{p\cos\theta + q\sin\theta}$
(2) $\dfrac{p^2 + q^2}{p^2\cos\theta + q^2\sin\theta}$
(3) $\dfrac{(p^2 + q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$
(4) $\dfrac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$

*Sol.* **(4)**




Using sine rule in triangle ABD
$$\frac{AB}{\sin\theta} = \frac{BD}{\sin(\theta + \alpha)}$$
$$\Rightarrow AB = \frac{\sqrt{p^2+q^2}\,\sin\theta}{\sin\theta\cos\alpha + \cos\theta\sin\alpha} = \frac{\sqrt{p^2+q^2}\,\sin\theta}{\frac{\sin\theta \cdot q}{\sqrt{p^2+q^2}} + \frac{\cos\theta \cdot p}{\sqrt{p^2+q^2}}}$$
$$\Rightarrow AB = \frac{(p^2+q^2)\sin\theta}{(p\cos\theta + q\sin\theta)}.$$

33. Given : A circle, $2x^2 + 2y^2 = 5$ and a parabola, $y^2 = 4\sqrt{5}\,x$.
**Statement – I :** An equation of a common tangent to these curves is $y = x + \sqrt{5}$.
**Statement – II :** If the line, $y = mx + \dfrac{\sqrt{5}}{m}$ $(m \neq 0)$ is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$.
(1) Statement - I is True; Statement -II is true; Statement-II is **not** a correct explanation for Statement-I
(2) Statement -I is True; Statement -II is False.
(3) Statement -I is False; Statement -II is True
(4) Statement -I is True; Statement -II is True; Statement-II is a **correct** explanation for Statement-I

*Sol.* **(1)**
Let the tangent to the parabola be $y = mx + \dfrac{\sqrt{5}}{m}$ $(m \neq 0)$.
Now, its distance from the centre of the circle must be equal to the radius of the circle.
So, $\left|\dfrac{\sqrt{5}}{m}\right| = \dfrac{\sqrt{5}}{\sqrt{2}}\sqrt{1+m^2} \Rightarrow (1 + m^2)\, m^2 = 2 \Rightarrow m^4 + m^2 - 2 = 0.$
$\Rightarrow (m^2 - 1)(m^2 + 2) = 0 \Rightarrow m = \pm 1$
So, the common tangents are $y = x + \sqrt{5}$ and $y = -x - \sqrt{5}$.

34. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected rays is

(1) $\sqrt{3}y = x - \sqrt{3}$ (2) $y = \sqrt{3}x - \sqrt{3}$

(3) $\sqrt{3}y = x - 1$ (4) $y = x + \sqrt{3}$

***Sol.*** **(1)**

Slope of the incident ray is $-\frac{1}{\sqrt{3}}$.

So, the slope of the reflected ray must be $\frac{1}{\sqrt{3}}$.

The point of incidence is $\left(\sqrt{3},\ 0\right)$. So, the equation of reflected ray is $y = \frac{1}{\sqrt{3}}\left(x - \sqrt{3}\right)$.

35. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given ?

(1) median (2) mode

(3) variance (4) mean

***Sol.*** **(3)**

Variance is not changed by the change of origin.

**Alternate Solution:**

$\sigma = \sqrt{\frac{\sum\left|x - \bar{x}\right|^2}{n}}$ for $y = x + 10 \Rightarrow \bar{y} = \bar{x} + 10$

$\sigma_1 = \sqrt{\frac{\sum\left|y + 10 - \bar{y} - 10\right|^2}{n}} = \sqrt{\frac{\sum\left|y - \bar{y}\right|^2}{n}} = \sigma$.

36. If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then

(1) $2x = 3y = 6z$ (2) $6x = 3y = 2z$

(3) $6x = 4y = 3z$ (4) $x = y = z$

***Sol.*** **(4)**

If x, y, z are in A.P.

$2y = x + z$

and $\tan^{-1}x$, $\tan^{-1}y$, $\tan^{-1}z$ are in A.P.

$2\tan^{-1}y = \tan^{-1}x + \tan^{-1}z \Rightarrow x = y = z$.

**Note: If y = 0, then none of the options is appropriate.**

37. If $\int f(x)\,dx = \Psi(x)$, then $\int x^5 f(x^3)\,dx$ is equal to

(1) $\frac{1}{3}x^3\Psi(x^3) - 3\int x^3\Psi(x^3)\,dx + C$ (2) $\frac{1}{3}x^3\Psi(x^3) - \int x^2\Psi(x^3)\,dx + C$

(3) $\frac{1}{3}\left[x^3\Psi(x^3) - \int x^3\Psi(x^3)\,dx\right] + C$ (4) $\frac{1}{3}\left[x^3\Psi(x^3) - \int x^2\Psi(x^3)\,dx\right] + C$

***Sol.*** **(2)**

$\int f(x)\,dx = \psi(x)$

Let $x^3 = t$

$3x^2dx = dt$

then $\int x^5 f(x^3)\,dx = \frac{1}{3}\int t f(t)\,dt$

$= \frac{1}{3}\left[t\int f(t)dt - \int\left\{1\cdot\int f(t)\,dt\right\}dt\right] = \frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)\,dx + C$.

38. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, and having centre at (0, 3) is

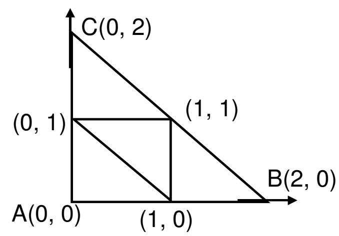
(1) $x^2+y^2-6y+7=0$
(2) $x^2+y^2-6y-5=0$
(3) $x^2+y^2-6y+5=0$
(4) $x^2+y^2-6y-7=0$

***Sol.*** **(4)**
foci ≡ (± ae, 0)
We have $a^2e^2 = a^2 - b^2 = 7$
Equation of circle $(x-0)^2 + (y-3)^2 = \left(\sqrt{7}-0\right)^2+\left(0-3\right)^2$
$\Rightarrow x^2 + y^2 - 6y - 7 = 0.$

39. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1) (1, 1) and (1, 0) is

(1) $2-\sqrt{2}$
(2) $1+\sqrt{2}$
(3) $1-\sqrt{2}$
(4) $2+\sqrt{2}$

***Sol.*** **(1)**




x-coordinate = $\frac{ax_1+bx_2+cx_3}{a+b+c}$
$= \frac{2\times2+2\sqrt{2}\times0+2\times0}{2+2+2\sqrt{2}}$
$= \frac{4}{4+2\sqrt{2}} = \frac{2}{2+\sqrt{2}} = 2-\sqrt{2}$.

**Alternate Solution:**
x-coordinate = r = (s – a) tan A/2
$= \left(\frac{4+2\sqrt{2}}{2}-2\sqrt{2}\right)\tan\frac{\pi}{4} = 2-\sqrt{2}$.

40. The intercepts on x-axis made by tangents to the curve, $y=\int_0^x |t|\,dt$, $x \in R$, which are parallel to the line y = 2x, are equal to

(1) ± 2
(2) ± 3
(3) ± 4
(4) ± 1

***Sol.*** **(4)**
$\frac{dy}{dx}=|x|=2 \Rightarrow x=\pm 2 \Rightarrow y=\int_0^2 |t|\,dt=2$ for x = 2
and y = $\int_0^{-2}|t|\,dt=-2$ for x = – 2
∴ tangents are y – 2 = 2 (x – 2) ⇒ y = 2x – 2
and y + 2 = 2 (x + 2) ⇒ y = 2x + 2
Putting y = 0, we get x = 1 and – 1.

41. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ….. , is

(1) $\frac{7}{9}\left(99-10^{-20}\right)$
(2) $\frac{7}{81}\left(179+10^{-20}\right)$
(3) $\frac{7}{9}\left(99+10^{-20}\right)$
(4) $\frac{7}{81}\left(179-10^{-20}\right)$

***Sol.*** **(2)**
$t_r$ = 0.777 ….. r times

$= 7\,(10^{-1} + 10^{-2} + 10^{-3} + \ldots.. + 10^{-r})$

$= \frac{7}{9}\left(1-10^{-r}\right)$

$$S_{20} = \sum_{r=1}^{20} t_r = \frac{7}{9}\left(20-\sum_{r=1}^{20}10^{-r}\right) = \frac{7}{9}\left(20-\frac{1}{9}\left(1-10^{-20}\right)\right) = \frac{7}{81}\left(179+10^{-20}\right)$$

42. Consider :

**Statement – I :** $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.

**Statement – II :** $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a tautology.

(1) Statement - I is True; Statement -II is true; Statement-II is **not** a correct explanation for Statement-I
(2) Statement -I is True; Statement -II is False.
(3) Statement -I is False; Statement -II is True
(4) Statement -I is True; Statement -II is True; Statement-II is a **correct** explanation for Statement-I

***Sol.*** **(1)**

S1:

| p | q | ~p | ~q | p^~q | ~p^q | (p^~q)^(~p^q) |
|---|---|---|---|---|---|---|
| T | T | F | F | F | F | F |
| T | F | F | T | T | F | F |
| F | T | T | F | F | T | F |
| F | F | T | T | F | F | F |

Fallacy

S2:

| p | q | ~ p | ~ q | $p \Rightarrow q$ | $\sim q \Rightarrow \sim p$ | $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ |
|---|---|---|---|---|---|---|
| T | T | F | F | T | T | T |
| T | F | F | T | F | F | T |
| F | T | T | F | T | T | T |
| F | F | T | T | T | T | T |

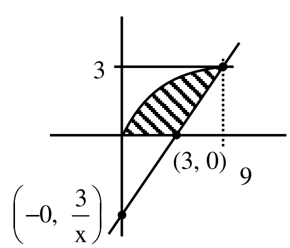
Tautology

S2 is not an explanation of S1

43. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis, and lying in the first quadrant is

(1) 36 (2) 18
(3) $\frac{27}{4}$ (4) 9

***Sol.*** **(4)**




$2\sqrt{x} = x - 3$

$4x = x^2 - 6x + 9$

$x^2 - 10x + 9$

$x = 9, x = 1$

$$\int_0^3 \left(2y+3\right) - y^2\,]dy$$

$$\left[y^2 + 3y - \frac{y^3}{3}\right]_0^3 = 9 + 9 - 9 = 9$$

44. The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as

(1) secA cosecA + 1 (2) tanA + cotA
(3) secA + cosecA (4) sinA cosA + 1

***Sol.*** **(1)**

$$\frac{1}{\cot A(1-\cot A)} - \frac{\cot^2 A}{(1-\cot A)} = \frac{1-\cot^3 A}{\cot A(1-\cot A)} = \frac{\operatorname{cosec}^2 A + \cot A}{\cot A} = 1 + \sec A \operatorname{cosec} A$$

45. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in [0, 1]

(1) lies between 2 and 3
(2) lies between −1 and 0
(3) does not exist
(4) lies between 1 and 2

***Sol.*** **(3)**

If $2x^3 + 3x + k = 0$ has 2 distinct real roots in [0, 1], then $f'(x)$ will change sign

but $f'(x) = 6x^2 + 3 > 0$

So no value of k exists.

46. $\lim_{x\to 0} \frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x}$ is equal to

(1) $\frac{1}{2}$
(2) 1
(3) 2
(4) $-\frac{1}{4}$

***Sol.*** **(3)**

$$\lim_{x\to 0} \frac{(1-\cos 2x)}{x(\tan 4x)}(3+\cos x)$$

$$\lim_{x\to 0} 2\left(\frac{\sin x}{x}\right)^2 \cdot \frac{1}{4}\left(\frac{4x}{\tan 4x}\right)(3+\cos x) = 2\times 1\times \frac{1}{4}\times 1\times(3+1) = 2.$$

47. Let $T_n$ be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is

(1) 5
(2) 10
(3) 8
(4) 7

***Sol.*** **(1)**

$^{n+1}C_3 - {}^nC_3 = 10 \Rightarrow {}^nC_2 = 10 \Rightarrow n = 5.$

48. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is

(1) 3000
(2) 3500
(3) 4500
(4) 2500

***Sol.*** **(2)**

$$\int_{2000}^{P} dP = \int_{0}^{25} (100 - 12\sqrt{x})\, dx$$

$$(P - 2000) = 25 \times 100 - \frac{12\times 2}{3}(25)^{3/2}$$

P = 3500.

49. **Statement – I :** The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$.

**Statement – II :** $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$.

(1) Statement - I is True; Statement -II is true; Statement-II is **not** a correct explanation for Statement-I
(2) Statement -I is True; Statement -II is False.

(3) Statement -I is False; Statement -II is True
(4) Statement -I is True; Statement -II is True; Statement-II is a **correct** explanation for Statement-I

***Sol.*** **(3)**

$$I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}\,dx$$

$$2I = \frac{\pi}{6}$$

$$I = \frac{\pi}{12}.$$

50. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a $3 \times 3$ matrix A and $|A| = 4$, then $\alpha$ is equal to

(1) 11
(2) 5
(3) 0
(4) 4

***Sol.*** **(1)**

$$P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$$

$|\text{Adj } A| = |A|^2$
$|\text{Adj } A| = 16$
$1\,(12 - 12) - \alpha\,(4 - 6) + 3\,(4 - 6) = 16.$
$2\alpha - 6 = 16.$
$2\alpha = 22.$
$\alpha = 11.$

51. The number of values of k, for which the system of equations

$(k + 1)x + 8y = 4k$
$kx + (k + 3)y = 3k - 1$

has no solution, is

(1) 1
(2) 2
(3) 3
(4) infinite

***Sol.*** **(1)**

For no solution

$$\frac{k+1}{k} = \frac{8}{k+3} \neq \frac{4k}{3k-1} \qquad \ldots (1)$$

$\Rightarrow (k + 1)(k + 3) - 8k = 0$
or $k^2 - 4k + 3 = 0 \Rightarrow k = 1, 3$
But for k = 1, equation (1) is not satisfied
Hence k = 3.

52. If $y = \sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at x = 1 is equal to

(1) $\frac{1}{2}$
(2) 1
(3) $\sqrt{2}$
(4) $\frac{1}{\sqrt{2}}$

***Sol.*** **(4)**

$y = \sec(\tan^{-1}x)$

$$\frac{dy}{dx} = \sec\left(\tan^{-1}x\right)\tan\left(\tan^{-1}x\right)\cdot\frac{1}{1+x^2}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \sqrt{2}\times 1\times\frac{1}{2} = \frac{1}{\sqrt{2}}.$$

53. If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, then k can have

(1) exactly one value
(2) exactly two values
(3) exactly three values
(4) any value

***Sol.*** **(2)**

$$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$1(1 + 2k) + 1(1 + k^2) - 1(2 - k) = 0$

$k^2 + 1 + 2k + 1 - 2 + k = 0$

$k^2 + 3k = 0$

$(k)(k + 3) = 0$

2 values of k.

54. Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is

(1) 220
(2) 219
(3) 211
(4) 256

***Sol.*** **(2)**

$A \times B$ will have 8 elements.

$2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2 = 256 - 1 - 8 - 28 = 219.$

55. If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is

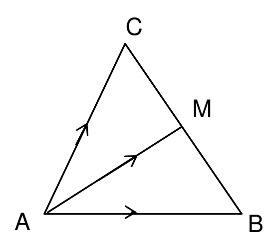
(1) $\sqrt{72}$
(2) $\sqrt{33}$
(3) $\sqrt{45}$
(4) $\sqrt{18}$

***Sol.*** **(2)**

$$\overrightarrow{AM} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$$

$\overrightarrow{AM} = 4\hat{i} - \hat{j} + 4\hat{k}$

$\left|\overrightarrow{AM}\right| = \sqrt{16+16+1} = \sqrt{33}$




56. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is

(1) $\frac{13}{3^5}$
(2) $\frac{11}{3^5}$
(3) $\frac{10}{3^5}$
(4) $\frac{17}{3^5}$

***Sol.*** **(2)**

P (correct answer) = 1/3

$${}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^1+{}^5C_5\left(\frac{1}{3}\right)^5$$

$$\frac{5\times2}{(3)^5}+\frac{1}{(3)^5}=\frac{11}{3^5}.$$

57. If z is a complex number of unit modulus and argument θ, then $\arg\left(\frac{1+z}{1+\bar{z}}\right)$ equals

(1) $\frac{\pi}{2}-\theta$ (2) $\theta$

(3) $\pi-\theta$ (4) $-\theta$

***Sol.*** **(2)**

$|z| = 1 \Rightarrow z\bar{z}=1$

$$\frac{1+z}{1+\bar{z}}=\frac{1+z}{1+\frac{1}{z}}=z.$$

58. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, a, b, c ∈ R, have a common root, then a : b : c is

(1) 3 : 2 : 1 (2) 1 : 3 : 2

(3) 3 : 1 : 2 (4) 1 : 2 : 3

***Sol.*** **(4)**

For equation $x^2 + 2x + 3 = 0$

both roots are imaginary.

Since a, b, c ∈ R.

If one root is common then both roots are common

Hence, $\frac{a}{1}=\frac{b}{2}=\frac{c}{3}$

a : b : c = 1 : 2 : 3.

59. Distance between two parallel planes 2x + y + 2z = 8 and 4x + 2y + 4z + 5 = 0 is

(1) $\frac{5}{2}$ (2) $\frac{7}{2}$

(3) $\frac{9}{2}$ (4) $\frac{3}{2}$

***Sol.*** **(2)**

4x + 2y + 4z = 16

4x + 2y + 4z = − 5

$d_{min}=\frac{21}{\sqrt{36}}=\frac{21}{6}=\frac{7}{2}.$

60. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is

(1) 120 (2) 210

(3) 310 (4) 4

***Sol.*** **(2)**

$$\left(\frac{\left(x^{1/3}+1\right)\left(x^{2/3}-x^{1/3}+1\right)}{x^{2/3}-x^{1/3}+1}-\frac{1}{\sqrt{x}}\cdot\frac{\left(\sqrt{x}+1\right)\left(\sqrt{x}-1\right)}{\left(\sqrt{x}-1\right)}\right)^{10}=(x^{1/3}-x^{-1/2})^{10}$$

$T_{r+1} = (-1)^r\,{}^{10}C_r\, x^{\frac{20-5r}{6}} \Rightarrow r = 4$

${}^{10}C_4 = 210.$