IIT-JEE 2012 Question Paper 2 Key

PART III : MATHEMATICS

SECTION I : Single Correct Answer Type

This section contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

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41. The equation of a plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3, 1, -1)$ is

(A) $5x - 11y + z = 17$ (B) $\sqrt{2}x + y = 3\sqrt{2} - 1$

(C) $x + y + z = \sqrt{3}$ (D) $x - \sqrt{2}y = 1 - \sqrt{2}$

ANSWER : A

42. Let PQR be a triangle of area Δ with $a = 2, b = \frac{7}{2}$ and $c = \frac{5}{2}$, where a, b and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R respectively. Then $\frac{2 \sin P - \sin 2P}{2 \sin P + \sin 2P}$ equals

(A) $\frac{3}{4\Delta}$ (B) $\frac{45}{4\Delta}$ (C) $\left(\frac{3}{4\Delta}\right)^2$ (D) $\left(\frac{45}{4\Delta}\right)^2$

ANSWER : C

43. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a} + \vec{b}| = \sqrt{29}$ and $\vec{a} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \vec{b}$, then a possible value of $(\vec{a} + \vec{b}) \cdot (-7\hat{i} + 2\hat{j} + 3\hat{k})$ is

(A) 0 (B) 3 (C) 4 (D) 8

ANSWER : C

IIT-JEE 2012 Question Paper 2 Key

MATHEMATICS

44. If P is a 3×3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3×3 identity matrix, then there exists a column matrix $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that

(A) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (B) $PX = X$ (C) $PX = 2X$ (D) $PX = -X$

ANSWER : D

45. Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation

$\left(\sqrt[3]{1+a}-1\right)x^2 + \left(\sqrt{1+a}-1\right)x + \left(\sqrt[6]{1+a}-1\right) = 0$ where $a > -1$.

Then $\lim_{a \to 0^+} \alpha(a)$ and $\lim_{a \to 0^+} \beta(a)$ are

(A) $-\frac{5}{2}$ and 1 (B) $-\frac{1}{2}$ and -1 (C) $-\frac{7}{2}$ and 2 (D) $-\frac{9}{2}$ and 3

ANSWER : B

46. Four fair dice D_1, D_2, D_3 and D_4, each having six faces numbered 1, 2, 3, 4, 5 and 6, are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is

(A) $\frac{91}{216}$ (B) $\frac{108}{216}$ (C) $\frac{125}{216}$ (D) $\frac{127}{216}$

ANSWER : A

47. The value of the integral

$$\int_{-\pi/2}^{\pi/2} \left(x^2 + \ln\frac{\pi+x}{\pi-x}\right)\cos x \, dx \text{ is}$$

(A) 0 (B) $\frac{\pi^2}{2} - 4$ (C) $\frac{\pi^2}{2} + 4$ (D) $\frac{\pi^2}{2}$

ANSWER : B

48. Let $a_1, a_2, a_3, \ldots$ be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is

(A) 22 (B) 23 (C) 24 (D) 25

ANSWER : D

IIT-JEE 2012 Question Paper 2 Key

MATHEMATICS

SECTION II : Paragraph Type

This section contains **6 multiple choice questions** relating to three paragraphs with **two questions on each paragraph.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

Paragraph for Questions 49 and 50

Let a_n denote the number of all n-digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let $b_n =$ the number of such n-digit integers ending with digit 1 and $c_n =$ the number of such n-digit integers ending with digit 0.

49. The value of b_6 is

(A) 7 (B) 8 (C) 9 (D) 11

ANSWER : B

50. Which of the following is correct?

(A) $a_{17} = a_{16} + a_{15}$ (B) $c_{17} \neq c_{16} + c_{15}$ (C) $b_{17} \neq b_{16} + c_{16}$ (D) $a_{17} = c_{17} + b_{16}$

ANSWER : A

IIT-JEE 2012 Question Paper 2 Key

MATHEMATICS

Paragraph for Questions 51 and 52

Let $f(x) = (1-x)^2 \sin^2 x + x^2$ for all $x \in \mathbb{R}$, and let $g(x) = \int_1^x \left(\frac{2(t-1)}{t+1} - \ln t\right) f(t)\, dt$ for all $x \in (1, \infty)$.

51. Which of the following is true?

(A) g is increasing on $(1, \infty)$

(B) g is decreasing on $(1, \infty)$

(C) g is increasing on $(1, 2)$ and decreasing on $(2, \infty)$

(D) g is decreasing on $(1, 2)$ and increasing on $(2, \infty)$

ANSWER : B

52. Consider the statements :

P : There exists some $x \in \mathbb{R}$ such that $f(x) + 2x = 2(1 + x^2)$

Q : There exists some $x \in \mathbb{R}$ such that $2f(x) + 1 = 2x(1 + x)$

Then

(A) both **P** and **Q** are true

(B) **P** is true and **Q** is false

(C) **P** is false and **Q** is true

(D) both **P** and **Q** are false

ANSWER : C

IIT-JEE 2012 Question Paper 2 Key

MATHEMATICS

Paragraph for Questions 53 and 54

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point $P(\sqrt{3}, 1)$. A straight line L, perpendicular to PT is a tangent to the circle $(x-3)^2 + y^2 = 1$.

53. A possible equation of L is

(A) $x - \sqrt{3}\,y = 1$ (B) $x + \sqrt{3}\,y = 1$ (C) $x - \sqrt{3}\,y = -1$ (D) $x + \sqrt{3}\,y = 5$

ANSWER : A

54. A common tangent of the two circles is

(A) $x = 4$ (B) $y = 2$ (C) $x + \sqrt{3}\,y = 4$ (D) $x + 2\sqrt{2}\,y = 6$

ANSWER : D

IIT-JEE 2012 Question Paper 2 Key

MATHEMATICS

SECTION III : Multiple Correct Answer(s) Type

This section contains **6 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct**.

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55. For every integer n, let a_n and b_n be real numbers. Let function $f: \mathbb{R} \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} a_n + \sin \pi x, & \text{for } x \in [2n, 2n+1] \\ b_n + \cos \pi x, & \text{for } x \in (2n-1, 2n) \end{cases}, \text{ for all integers } n.$$

If f is continuous, then which of the following hold(s) for all n?

(A) $a_{n-1} - b_{n-1} = 0$ (B) $a_n - b_n = 1$ (C) $a_n - b_{n+1} = 1$ (D) $a_{n-1} - b_n = -1$

ANSWER : BD

56. If $f(x) = \int_0^x e^{t^2} (t-2)(t-3)\, dt$ for all $x \in (0, \infty)$, then

(A) f has a local maximum at $x = 2$

(B) f is decreasing on (2, 3)

(C) there exists some $c \in (0, \infty)$ such that $f''(c) = 0$

(D) f has a local minimum at $x = 3$

ANSWER : ABCD

IIT-JEE 2012 Question Paper 2 Key

MATHEMATICS

57. If the straight lines $\frac{x-1}{2}=\frac{y+1}{k}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{k}$ are coplanar, then the plane(s) containing these two lines is(are)

(A) $y+2z=-1$ (B) $y+z=-1$ (C) $y-z=-1$ (D) $y-2z=-1$

ANSWER : BC

58. Let X and Y be two events such that $P(X \mid Y)=\frac{1}{2}$, $P(Y \mid X)=\frac{1}{3}$ and $P(X \cap Y)=\frac{1}{6}$. Which of the following is (are) correct?

(A) $P(X \cup Y)=\frac{2}{3}$

(B) X and Y are independent

(C) X and Y are not independent

(D) $P(X^{c} \cap Y)=\frac{1}{3}$

ANSWER : AB

IIT-JEE 2012 Question Paper 2 Key

MATHEMATICS

59. If the adjoint of a 3×3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of P is (are)

(A) -2 (B) -1 (C) 1 (D) 2

ANSWER : AD

60. Let $f:(-1, 1)\rightarrow \mathbb{R}$ be such that $f(\cos 4\theta) = \dfrac{2}{2-\sec^2\theta}$ for $\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then the value(s) of $f\left(\frac{1}{3}\right)$ is (are)

(A) $1-\sqrt{\frac{3}{2}}$ (B) $1+\sqrt{\frac{3}{2}}$ (C) $1-\sqrt{\frac{2}{3}}$ (D) $1+\sqrt{\frac{2}{3}}$

Zero Marks to all