PART III : MATHEMATICS

SECTION – I (Total Marks : 24)

(Single Correct Answer Type)

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

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41. Let $P(6,3)$ be a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the normal at the point P intersects the $x-$axis at $(9, 0)$, then the eccentricity of the hyperbola is

(A) $\sqrt{\frac{5}{2}}$ (B) $\sqrt{\frac{3}{2}}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$

ANSWER : B

42. A value of b for which the equations

$$x^2+bx-1=0$$
$$x^2+x+b=0,$$

have one root in common is

(A) $-\sqrt{2}$ (B) $-i\sqrt{3}$ (C) $i\sqrt{5}$ (D) $\sqrt{2}$

ANSWER : B

43. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form

$$\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix},$$

where each of a, b, and c is either ω or ω^2. Then the number of distinct matrices in the set S is

(A) 2 (B) 6 (C) 4 (D) 8

ANSWER : A

44. The circle passing through the point $(-1, 0)$ and touching the $y-$axis at $(0, 2)$ also passes through the point

(A) $\left(-\frac{3}{2}, 0\right)$ (B) $\left(-\frac{5}{2}, 2\right)$ (C) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (D) $(-4, 0)$

ANSWER : D

45. If

$$\lim_{x \to 0} \left[1 + x\ln(1+b^2)\right]^{\frac{1}{x}} = 2b\sin^2\theta,\ b > 0 \text{ and } \theta \in (-\pi, \pi],$$

then the value of θ is

(A) $\pm\frac{\pi}{4}$ (B) $\pm\frac{\pi}{3}$ (C) $\pm\frac{\pi}{6}$ (D) $\pm\frac{\pi}{2}$

ANSWER : D

46. Let $f : [-1, 2] \to [0, \infty)$ be a continuous function such that $f(x) = f(1-x)$ for all $x \in [-1, 2]$. Let $R_1 = \int_{-1}^{2} x f(x)\, dx$, and R_2 be the area of the region bounded by $y = f(x)$, $x = -1$, $x = 2$, and the x-axis. Then

(A) $R_1 = 2R_2$ (B) $R_1 = 3R_2$ (C) $2R_1 = R_2$ (D) $3R_1 = R_2$

ANSWER : C

47. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in \mathbb{R}$. Then the set of all x satisfying $(f \circ g \circ g \circ f)(x) = (g \circ g \circ f)(x)$, where $(f \circ g)(x) = f(g(x))$, is

(A) $\pm\sqrt{n\pi}$, $n \in \{0, 1, 2, ...\}$ (B) $\pm\sqrt{n\pi}$, $n \in \{1, 2, ...\}$

(C) $\frac{\pi}{2} + 2n\pi$, $n \in \{..., -2, -1, 0, 1, 2, ...\}$ (D) $2n\pi$, $n \in \{..., -2, -1, 0, 1, 2, ...\}$

ANSWER : A

48. Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from $(0, 0)$ to (x, y) in the ratio $1:3$. Then the locus of P is

(A) $x^2 = y$ (B) $y^2 = 2x$ (C) $y^2 = x$ (D) $x^2 = 2y$

ANSWER : C

SECTION – II (Total Marks : 16)

(Multiple Correct Answer(s) Type)

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** may be correct.

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49. If

$$f(x)=\begin{cases} -x-\frac{\pi}{2}, & x\le -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2}<x\le 0 \\ x-1, & 0<x\le 1 \\ \ln x, & x>1, \end{cases}$$

then

(A) $f(x)$ is continuous at $x=-\frac{\pi}{2}$

(B) $f(x)$ is not differentiable at $x=0$

(C) $f(x)$ is differentiable at $x=1$

(D) $f(x)$ is differentiable at $x=-\frac{3}{2}$

ANSWER : ABCD

50. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If $P(T)$ denotes the probability of occurrence of the event T, then

(A) $P(E)=\frac{4}{5},\ P(F)=\frac{3}{5}$

(B) $P(E)=\frac{1}{5},\ P(F)=\frac{2}{5}$

(C) $P(E)=\frac{2}{5},\ P(F)=\frac{1}{5}$

(D) $P(E)=\frac{3}{5},\ P(F)=\frac{4}{5}$

ANSWER : AD

51. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point $(9, 6)$, then L is given by

(A) $y - x + 3 = 0$ (B) $y + 3x - 33 = 0$

(C) $y + x - 15 = 0$ (D) $y - 2x + 12 = 0$

ANSWER : ABD

52. Let $f : (0,1) \to \mathbb{R}$ be defined by

$$f(x) = \frac{b - x}{1 - bx},$$

where b is a constant such that $0 < b < 1$. Then

(A) f is not invertible on $(0, 1)$

(B) $f \neq f^{-1}$ on $(0, 1)$ and $f'(b) = \frac{1}{f'(0)}$

(C) $f = f^{-1}$ on $(0, 1)$ and $f'(b) = \frac{1}{f'(0)}$

(D) f^{-1} is differentiable on $(0, 1)$

ANSWER : A

SECTION – III (Total Marks : 24)

(Integer Answer Type)

This section contains **6 questions**. The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

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53. Let $\omega = e^{i\pi/3}$, and a, b, c, x, y, z be non-zero complex numbers such that

$$a + b + c = x$$
$$a + b\omega + c\omega^2 = y$$
$$a + b\omega^2 + c\omega = z.$$

Then the value of $\dfrac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$ is

ANSWER : MARKS TO ALL

54. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is

ANSWER : 2

55. Let $y'(x) + y(x)g'(x) = g(x)g'(x)$, $y(0) = 0$, $x \in \mathbb{R}$, where $f'(x)$ denotes $\dfrac{d\,f(x)}{dx}$ and $g(x)$ is a given non-constant differentiable function on $\mathbb{R}$ with $g(0) = g(2) = 0$. Then the value of $y(2)$ is

ANSWER : 0

56. Let M be a 3×3 matrix satisfying

$$M\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix},\quad M\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix},\text{ and }M\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}.$$

Then the sum of the diagonal entries of M is

ANSWER : 9

57. Let $\vec{a}=-\hat{i}-\hat{k}$, $\vec{b}=-\hat{i}+\hat{j}$ and $\vec{c}=\hat{i}+2\hat{j}+3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}\,.\,\vec{a}=0$, then the value of $\vec{r}.\vec{b}$ is

ANSWER : 9

58. The straight line $2x-3y=1$ divides the circular region $x^2+y^2\le 6$ into two parts. If

$$S=\left\{\left(2,\frac{3}{4}\right),\left(\frac{5}{2},\frac{3}{4}\right),\left(\frac{1}{4},-\frac{1}{4}\right),\left(\frac{1}{8},\frac{1}{4}\right)\right\},$$

then the number of point(s) in S lying inside the smaller part is

ANSWER : 2

SECTION – IV (Total Marks : 16)

(Matrix-Match Type)

This section contains **2 questions**. Each question has **four statements** (A, B, C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II**. Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

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59. Match the statements given in **Column I** with the values given in **Column II**

Column I	Column II
(A) If $\vec{a} = \hat{j} + \sqrt{3}\,\hat{k}$, $\vec{b} = -\hat{j} + \sqrt{3}\,\hat{k}$ and $\vec{c} = 2\sqrt{3}\,\hat{k}$ form a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is	(p) $\frac{\pi}{6}$
(B) If $\int_a^b (f(x) - 3x)dx = a^2 - b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is	(q) $\frac{2\pi}{3}$
(C) The value of $\frac{\pi^2}{\ln 3} \int_{7/6}^{5/6} \sec(\pi x)\, dx$ is	(r) $\frac{\pi}{3}$
(D) The maximum value of $\left\lvert Arg\left(\frac{1}{1-z}\right)\right\rvert$ for $\lvert z \rvert = 1,\ z \neq 1$ is given by	(s) π
	(t) $\frac{\pi}{2}$

ANSWER A : q
B : p or p, q, r, s and t
C : s
D : t

60. Match the statements given in **Column I** with the intervals/union of intervals given in **Column II**

Column I	Column II
(A) The set $\left\{ \mathrm{Re}\left(\frac{2iz}{1-z^2} \right) : z \text{ is a complex number}, \lvert z \rvert = 1, z \neq \pm 1 \right\}$ is	(p) $(-\infty, -1) \cup (1, \infty)$
(B) The domain of the function $f(x) = \sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}} \right)$ is	(q) $(-\infty, 0) \cup (0, \infty)$
(C) If $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$, then the set $\left\{ f(\theta) : 0 \leq \theta < \frac{\pi}{2} \right\}$ is	(r) $[2, \infty)$
(D) If $f(x) = x^{3/2}(3x-10)$, $x \geq 0$, then $f(x)$ is increasing in	(s) $(-\infty, -1] \cup [1, \infty)$
	(t) $(-\infty, 0] \cup [2, \infty)$

ANSWER A : s
B : t
C : r
D : r