PART III : MATHEMATICS

SECTION – I (Total Marks : 21)

(Single Correct Answer Type)

This section contains **7 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

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47. Let (x_0, y_0) be the solution of the following equations

$$(2x)^{\ln 2} = (3y)^{\ln 3}$$
$$3^{\ln x} = 2^{\ln y}.$$

Then x_0 is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 6

ANSWER : C

48. The value of $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)} \, dx$ is

(A) $\frac{1}{4}\ln\frac{3}{2}$ (B) $\frac{1}{2}\ln\frac{3}{2}$

(C) $\ln\frac{3}{2}$ (D) $\frac{1}{6}\ln\frac{3}{2}$

ANSWER : A

49. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is given by

(A) $\hat{i} - 3\hat{j} + 3\hat{k}$ (B) $-3\hat{i} - 3\hat{j} - \hat{k}$

(C) $3\hat{i} - \hat{j} + 3\hat{k}$ (D) $\hat{i} + 3\hat{j} - 3\hat{k}$

ANSWER: C

50. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then

(A) $P \subset Q$ and $Q - P \neq \varnothing$ (B) $Q \not\subset P$

(C) $P \not\subset Q$ (D) $P = Q$

ANSWER: D

51. Let the straight line $x = b$ divide the area enclosed by $y = (1-x)^2$, $y = 0$, and $x = 0$ into two parts R_1 $(0 \le x \le b)$ and R_2 $(b \le x \le 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then b equals

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

ANSWER: B

52. Let α and β be the roots of $x^2-6x-2=0$, with $\alpha>\beta$. If $a_n=\alpha^n-\beta^n$ for $n\geq 1$, then the value of $\frac{a_{10}-2a_8}{2a_9}$ is

(A) 1 (B) 2 (C) 3 (D) 4

ANSWER : C

53. A straight line L through the point $(3,-2)$ is inclined at an angle $60°$ to the line $\sqrt{3}x+y=1$. If L also intersects the x-axis, then the equation of L is

(A) $y+\sqrt{3}x+2-3\sqrt{3}=0$

(B) $y-\sqrt{3}x+2+3\sqrt{3}=0$

(C) $\sqrt{3}y-x+3+2\sqrt{3}=0$

(D) $\sqrt{3}y+x-3+2\sqrt{3}=0$

ANSWER : B

SECTION – II (Total Marks : 16)

(Multiple Correct Answers Type)

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** may be correct.

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54. The vector(s) which is/are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$, and perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ is/are

(A) $\hat{j}-\hat{k}$ (B) $-\hat{i}+\hat{j}$ (C) $\hat{i}-\hat{j}$ (D) $-\hat{j}+\hat{k}$

ANSWER : AD

55. Let M and N be two 3×3 non-singular skew-symmetric matrices such that $MN=NM$. If P^T denotes the transpose of P, then $M^2N^2(M^TN)^{-1}(MN^{-1})^T$ is equal to

(A) M^2 (B) $-N^2$ (C) $-M^2$ (D) MN

ANSWER : MARKS TO ALL

56. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be reciprocal to that of the ellipse $x^2+4y^2=4$. If the hyperbola passes through a focus of the ellipse, then

(A) the equation of the hyperbola is $\frac{x^2}{3}-\frac{y^2}{2}=1$

(B) a focus of the hyperbola is $(2, 0)$

(C) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$

(D) the equation of the hyperbola is $x^2-3y^2=3$

ANSWER : BD

57. Let $f:\mathbb{R}\rightarrow\mathbb{R}$ be a function such that

$$f(x+y)=f(x)+f(y),\quad \forall x,y\in\mathbb{R}.$$

If $f(x)$ is differentiable at $x=0$, then

(A) $f(x)$ is differentiable only in a finite interval containing zero

(B) $f(x)$ is continuous $\forall x\in\mathbb{R}$

(C) $f'(x)$ is constant $\forall x\in\mathbb{R}$

(D) $f(x)$ is differentiable except at finitely many points

ANSWER : BC, BCD

SECTION – III (Total Marks : 15)

(Paragraph Type)

This section contains **2 paragraphs**. Based upon one of the paragraphs **3 multiple choice questions** and based on the other paragraph **2 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 58 to 60

Let a, b and c be three real numbers satisfying

$$[a \quad b \quad c]\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = [0 \quad 0 \quad 0] \qquad \text{...........(E)}$$

58. If the point $P(a, b, c)$, with reference to (E), lies on the plane $2x + y + z = 1$, then the value of $7a + b + c$ is

(A) 0 (B) 12 (C) 7 (D) 6

ANSWER : D

59. Let ω be a solution of $x^3 - 1 = 0$ with $\text{Im}(\omega) > 0$. If $a = 2$ with b and c satisfying (E), then the value of

$$\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c}$$

is equal to

(A) -2 (B) 2 (C) 3 (D) -3

ANSWER : A

60. Let $b = 6$, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then

$$\sum_{n=0}^{\infty}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^n$$

is

(A) 6 (B) 7 (C) $\frac{6}{7}$ (D) ∞

ANSWER : B

Paragraph for Question Nos. 61 and 62

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2.

61. The probability of the drawn ball from U_2 being white is

(A) $\frac{13}{30}$ (B) $\frac{23}{30}$ (C) $\frac{19}{30}$ (D) $\frac{11}{30}$

ANSWER : B

62. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(A) $\frac{17}{23}$ (B) $\frac{11}{23}$ (C) $\frac{15}{23}$ (D) $\frac{12}{23}$

ANSWER : D

SECTION – IV (Total Marks : 28)

(Integer Answer Type)

This section contains **7 questions**. The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

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63. Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola, and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum. Then $\frac{\Delta_1}{\Delta_2}$ is

ANSWER : 2

64. Let $a_1, a_2, a_3, \ldots, a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i$, $1 \le p \le 100$. For any integer n with $1 \le n \le 20$, let $m = 5n$. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is

ANSWER : 3, 9, 3 & 9 BOTH

65. The positive integer value of $n > 3$ satisfying the equation

$$\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$$

is

ANSWER : 7

66. Let $f:[1,\infty)\to[2,\infty)$ be a differentiable function such that $f(1)=2$. If

$$6\int_1^x f(t)\,dt = 3x\,f(x) - x^3$$

for all $x \geq 1$, then the value of $f(2)$ is

ANSWER : MARKS TO ALL

67. If z is any complex number satisfying $|z-3-2i| \leq 2$, then the minimum value of $|2z-6+5i|$ is

ANSWER : 5

68. The minimum value of the sum of real numbers a^{-5}, a^{-4}, $3a^{-3}$, $1, a^8$ and a^{10} with $a > 0$ is

ANSWER : 8

69. Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$. Then the value of

$$\frac{d}{d(\tan\theta)}\left(f(\theta)\right)$$

is

ANSWER : 1