JEE MAIN

2010

PART – II : MATHEMATICS

SECTION – I (Single Correct Choice Type)

20. For r = 0, 1, ..., 10, let A_r, B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1+x)^{10}$, $(1+x)^{20}$ and $(1+x)^{30}$. Then

$$\sum_{r=1}^{10} A_r\,(B_{10}B_r - C_{10}A_r)$$

is equal to

A) $B_{10} - C_{10}$ B) $A_{10}\left(B_{10}^2 - C_{10}A_{10}\right)$ C) 0 D) $C_{10} - B_{10}$

ANSWER: D

21. Let S = {1, 2, 3, 4}. The total number of unordered pairs of disjoint subsets of S is equal to

A) 25 B) 34 C) 42 D) 41

ANSWER: D

22. Let f be a real-valued function defined on the interval (–1, 1) such that

$e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4 + 1}\, dt$, for all $x \in (-1, 1)$, and let f^{-1} be the inverse function of f.

Then $(f^{-1})'(2)$ is equal to

A) 1 B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{1}{e}$

ANSWER: B

23. If the distance of the point P(1, –2, 1) from the plane x + 2y – 2z = α, where $\alpha > 0$, is 5, then the foot of the perpendicular from P to the plane is

A) $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$ B) $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$ C) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$ D) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$

ANSWER: A

24. Two adjacent sides of a parallelogram ABCD are given by

$\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$

The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by

A) $\frac{8}{9}$ B) $\frac{\sqrt{17}}{9}$ C) $\frac{1}{9}$ D) $\frac{4\sqrt{5}}{9}$

ANSWER: B

25. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is

A) $\frac{3}{5}$ B) $\frac{6}{7}$ C) $\frac{20}{23}$ D) $\frac{9}{20}$

ANSWER: C

SECTION – II (Integer Type)

26. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where $k > 0$, then the value of [k] is

[**Note** : [k] denotes the largest integer less than or equal to k]

ANSWER: 3

27. Consider a triangle ABC and let a, b and c denote the lengths of the sides opposite to vertices A, B and C respectively. Suppose a = 6, b = 10 and the area of the triangle is $15\sqrt{3}$. If $\angle ACB$ is obtuse and if r denotes the radius of the incircle of the triangle, then r^2 is equal to

ANSWER: 3

28. Let f be a function defined on **R** (the set of all real numbers) such that

$f'(x) = 2010(x-2009)(x-2010)^2(x-2011)^3(x-2012)^4$, for all $x \in$ **R**.

If g is a function defined on **R** with values in the interval $(0, \infty)$ such that

$$f(x) = \ell n\ (g(x)), \text{ for all } x \in \mathbf{R},$$

then the number of points in **R** at which g has a local maximum is

ANSWER: 1

29. Let $a_1, a_2, a_3, \ldots, a_{11}$ be real numbers satisfying

$a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, \ldots, 11$.

If $\frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + \ldots + a_{11}}{11}$ is equal to

ANSWER: 0

30. Let k be a positive real number and let

$$A=\begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} \text{ and } B=\begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}.$$

If det (adj A) + det(adj B) $=10^6$, then [k] is equal to

[**Note** : adj M denotes the adjoint of a square matrix M and [k] denotes the largest integer less than or equal to k].

ANSWER: 4

SECTION – III (Paragraph Type)

Paragraph for questions 31 to 33.

Consider the polynomial

$$f(x)=1+2x+3x^2+4x^3.$$

Let s be the sum of all distinct real roots of f(x) and let $t = |s|$.

31. The real number s lies in the interval

A) $\left(-\frac{1}{4},0\right)$ B) $\left(-11,-\frac{3}{4}\right)$ C) $\left(-\frac{3}{4},-\frac{1}{2}\right)$ D) $\left(0,\frac{1}{4}\right)$

ANSWER: C

32. The area bounded by the curve y = f(x) and the lines x = 0, y = 0 and x = t, lies in the interval

A) $\left(\frac{3}{4},3\right)$ B) $\left(\frac{21}{64},\frac{11}{16}\right)$ C) $(9,10)$ D) $\left(0,\frac{21}{64}\right)$

ANSWER: A

33. The function $f'(x)$ is

A) increasing in $\left(-t,-\frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4},t\right)$

B) decreasing in $\left(-t,-\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4},t\right)$

C) increasing in $(-t,\ t)$

D) decreasing in $(-t,\ t)$

ANSWER: B

Paragraph for Questions 34 to 36.

Tangents are drawn from the point P(3, 4) to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ touching the ellipse at points A and B.

34. The coordinates of A and B are

A) (3, 0) and (0, 2)

B) $\left(-\frac{8}{5},\frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5},\frac{8}{5}\right)$

C) $\left(-\frac{8}{5},\frac{2\sqrt{161}}{15}\right)$ and (0, 2)

D) (3, 0) and $\left(-\frac{9}{5},\frac{8}{5}\right)$

ANSWER: D

35. The orthocenter of the triangle PAB is

A) $\left(5,\frac{8}{7}\right)$

B) $\left(\frac{7}{5},\frac{25}{8}\right)$

C) $\left(\frac{11}{5},\frac{8}{5}\right)$

D) $\left(\frac{8}{25},\frac{7}{5}\right)$

ANSWER: C

36. The equation of the locus of the point whose distances from the point P and the line AB are equal, is

A) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

B) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$

C) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$

D) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

ANSWER: A

SECTION – IV (Matrix Type)

37. Match the statements in **Column-I** with those in **Column-II**.

[**Note**: Here z takes values in the complex plane and Im z and Re z denote, respectively, the imaginary part and the real part of z.]

Column I	Column II
A) The set of points z satisfying $\lvert z - i\lvert z\rvert\rvert = \lvert z + i\lvert z\rvert\rvert$ is contained in or equal to	p) an ellipse with eccentricity $\frac{4}{5}$
	q) the set of points z satisfying Im z = 0
B) The set of points z satisfying $\lvert z + 4\rvert + \lvert z - 4\rvert = 10$ is contained in or equal to	r) the set of points z satisfying $\lvert \text{Im } z\rvert \le 1$
C) If $\lvert w\rvert = 2$, then the set of points $z = w - \frac{1}{w}$ is contained in or equal to	s) the set of points z satisfying $\lvert \text{Re } z\rvert \le 2$
D) If $\lvert w\rvert = 1$, then the set of points $z = w + \frac{1}{w}$ is contained in or equal to	t) the set of points z satisfying $\lvert z\rvert \le 3$

ANSWER:

A: q and r
B: p
C: p and s and t
D: q and r and s and t

38. Match the statements in **Column-I** with the values in **Column-II**.

Column I	Column II
A) A line from the origin meets the lines $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ and $\frac{x-\frac{8}{3}}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$ at P and Q respectively. If length PQ = d, then d^2 is	p) – 4
B) The values of x satisfying $\tan^{-1}(x+3)-\tan^{-1}(x-3)=\sin^{-1}\left(\frac{3}{5}\right)$ are	q) 0
C) Non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy $\vec{a}.\vec{b}=0$, $(\vec{b}-\vec{a}).(\vec{b}+\vec{c})=0$ and $2\lvert\vec{b}+\vec{c}\rvert=\lvert\vec{b}-\vec{a}\rvert$. If $\vec{a}=\mu\vec{b}+4\vec{c}$, then the possible values of μ are	r) 4
D) Let f be the function on $[-\pi,\pi]$ given by f(0) = 9 and f(x) = $\sin\left(\frac{9x}{2}\right)\Big/\sin\left(\frac{x}{2}\right)$ for $x \neq 0$. The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$ is	s) 5
	t) 6

ANSWER:

A: t

B: p and r

C: either q or (q and s)

D: r