

PART B – MATHEMATICS

31. **Statement-1** : The sum of the series $1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + \dots + (361 + 380 + 400)$ is 8000.

Statement-2 : $\sum_{k=1}^n (k^3 - (k-1)^3) = n^3$, for any

natural number n.

- (1) Statement-1 is true, Statement-2 is false.
- (2) Statement-1 is false, Statement-2 is true.
- (3) Statement-1 is true, Statement-2 is true ; Statement-2 is a correct explanation for Statement-1.
- (4) Statement-1 is true, Statement-2 is true ; Statement-2 is not a correct explanation for Statement-1.

Ans. (3)

32. An ellipse is drawn by taking a diameter of the circle $(x - 1)^2 + y^2 = 1$ as its semi-minor axis and a diameter of the circle $x^2 + (y - 2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is :

- (1) $x^2 + 4y^2 = 16$
- (2) $4x^2 + y^2 = 4$
- (3) $x^2 + 4y^2 = 8$
- (4) $4x^2 + y^2 = 8$

Ans. (1)

33. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is :

- (1) 5/3
- (2) 10/3
- (3) 3/5
- (4) 6/5

Ans. (2)

34. Let P and Q be 3×3 matrices with $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to :

- (1) -1
- (2) -2
- (3) 1
- (4) 0

Ans. (4)

35. If n is a positive integer, then

$(\sqrt{3} + 1)^{2n} - (\sqrt{3} - 1)^{2n}$ is :

- (1) a rational number other than positive integers
- (2) an irrational number
- (3) an odd positive integer
- (4) an even positive integer

Ans. (2)

36. **Statement-1** : An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$.

Statement-2 : If the line $y = mx + \frac{4\sqrt{3}}{m}$,

($m \neq 0$) is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$.

- (1) Statement-1 is true, Statement-2 is false.
- (2) Statement-1 is false, Statement-2 is true.
- (3) Statement-1 is true, Statement-2 is true ; Statement-2 is a correct explanation for Statement-1.
- (4) Statement-1 is true, Statement-2 is true ; Statement-2 is not a correct explanation for Statement-1.

Ans. (3)

37. Three numbers are chosen at random without replacement from $\{1, 2, 3, \dots, 8\}$. The probability that their minimum is 3, given that their maximum is 6, is :

- (1) $\frac{2}{5}$
- (2) $\frac{3}{8}$
- (3) $\frac{1}{5}$
- (4) $\frac{1}{4}$

Ans. (3)

38. If $g(x) = \int_0^x \cos 4t \, dt$, then $g(x + \pi)$ equals :

- (1) $g(x) \cdot g(\pi)$
- (2) $\frac{g(x)}{g(\pi)}$
- (3) $g(x) + g(\pi)$
- (4) $g(x) - g(\pi)$

Ans. (3 & 4)

39. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is :-

- (1) 879
- (2) 880
- (3) 629
- (4) 630

Ans. (1)

40. If 100 times the 100th term of an A.P. with non-zero common difference equals the 50 times its 50th term, then the 150th term of this A.P. is :

- (1) zero
- (2) -150
- (3) 150 times its 50th term
- (4) 150

Ans. (1)

41. The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$, and the straight line $y = 2$ is :

- (1) $10\sqrt{2}$ (2) $20\sqrt{2}$ (3) $\frac{10\sqrt{2}}{3}$ (4) $\frac{20\sqrt{2}}{3}$

Ans. (4)

42. An equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is :

- (1) $x - 2y + 2z + 5 = 0$
 (2) $x - 2y + 2z - 3 = 0$
 (3) $x - 2y + 2z + 1 = 0$
 (4) $x - 2y + 2z - 1 = 0$

Ans. (2)

43. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has :

- (1) exactly four real roots.
 (2) infinite number of real roots.
 (3) no real roots.
 (4) exactly one real root.

Ans. (3)

44. The negation of the statement "If I become a teacher, then I will open a school", is :

- (1) I will not become a teacher or I will open a school.
 (2) I will become a teacher and I will not open a school.
 (3) Either I will not become a teacher or I will not open a school.
 (4) Neither I will become a teacher nor I will open a school.

Ans. (2)

45. The population $p(t)$ at time t of a certain mouse species satisfies the differential equation

$\frac{dp(t)}{dt} = 0.5 p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is:

- (1) $\ln 18$ (2) $2 \ln 18$ (3) $\ln 9$ (4) $\frac{1}{2} \ln 18$

Ans. (2)

46. If the integral

$$\int \frac{5 \tan x}{\tan x - 2} dx = x + a \ln |\sin x - 2 \cos x| + k$$

then a is equal to :

- (1) 2 (2) -1 (3) -2 (4) 1

Ans. (1)

47. Let \hat{a} and \hat{b} be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then the angle between \hat{a} and \hat{b} is :

- (1) $\frac{\pi}{4}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{3}$

Ans. (4)

48. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is :

- (1) $-\frac{1}{2}$ (2) $-\frac{1}{4}$ (3) -4 (4) -2

Ans. (4)

49. Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is :

- (1) 5^3 (2) 5^2 (3) 3^5 (4) 2^5

Ans. (3)

50. Let ABCD be a parallelogram such that $\vec{AB} = \vec{q}$, $\vec{AD} = \vec{p}$ and $\angle BAD$ be an acute angle.

If \vec{r} is the vector that coincides with the altitude directed from the vertex B to the side AD, then \vec{r} is given by :

(1) $\vec{r} = -3\vec{q} + \frac{3(\vec{p} \cdot \vec{q})}{(\vec{p} \cdot \vec{p})}\vec{p}$

(2) $\vec{r} = 3\vec{q} - \frac{3(\vec{p} \cdot \vec{q})}{(\vec{p} \cdot \vec{p})}\vec{p}$

(3) $\vec{r} = -\vec{q} + \left(\frac{\vec{p} \cdot \vec{q}}{\vec{p} \cdot \vec{p}}\right)\vec{p}$

(4) $\vec{r} = \vec{q} - \left(\frac{\vec{p} \cdot \vec{q}}{\vec{p} \cdot \vec{p}}\right)\vec{p}$

Ans. (3)

51. If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals :

- (1) $\frac{11}{5}$ (2) $\frac{29}{5}$ (3) 5 (4) 6

Ans. (4)

52. In a ΔPQR , if $3 \sin P + 4 \cos Q = 6$ and $4 \sin Q + 3 \cos P = 1$, then the angle R is equal to :

- (1) $\frac{3\pi}{4}$ (2) $\frac{5\pi}{6}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{4}$

Ans. (3)

53. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column

matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$,

then $u_1 + u_2$ is equal to :

- (1) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ (2) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (3) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ (4) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$

Ans. (1)

54. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is a function defined by

$$f(x) = [x] \cos\left(\frac{2x-1}{2}\right)\pi, \text{ where } [x] \text{ denotes the}$$

greatest integer function, then f is :

- (1) continuous only at $x = 0$.
 (2) continuous for every real x.
 (3) discontinuous only at $x = 0$.
 (4) discontinuous only at non-zero integral values of x.

Ans. (2)

55. A spherical balloon is filled with 4500π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is :

- (1) $9/2$ (2) $9/7$ (3) $7/9$ (4) $2/9$

Ans. (4)

56. Let $a, b \in \mathbb{R}$ be such that the function f given by $f(x) = \ln|x| + bx^2 + ax$, $x \neq 0$ has extreme values at $x = -1$ and $x = 2$.

Statement-1 : f has local maximum at $x = -1$ and at $x = 2$.

Statement-2 : $a = \frac{1}{2}$ and $b = \frac{-1}{4}$.

- (1) Statement-1 is true, Statement-2 is false.
 (2) Statement-1 is false, Statement-2 is true.
 (3) Statement-1 is true, Statement-2 is true ; Statement-2 is a correct explanation for Statement-1.
 (4) Statement-1 is true, Statement-2 is true ; Statement-2 is not a correct explanation for Statement-1.

Ans. (3)

57. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies :

- (1) on the imaginary axis.
 (2) either on the real axis or on a circle passing through the origin.
 (3) on a circle with centre at the origin.
 (4) either on the real axis or on a circle not passing through the origin.

Ans. (2)

58. Consider the function, $f(x) = |x-2| + |x-5|$, $x \in \mathbb{R}$.

Statement-1 : $f'(4) = 0$.

Statement-2 : f is continuous in $[2, 5]$, differentiable in $(2, 5)$ and $f(2) = f(5)$.

- (1) Statement-1 is true, Statement-2 is false.
 (2) Statement-1 is false, Statement-2 is true.
 (3) Statement-1 is true, Statement-2 is true ; Statement-2 is a correct explanation for Statement-1.
 (4) Statement-1 is true, Statement-2 is true ; Statement-2 is not a correct explanation for Statement-1.

Ans. (4)

59. Let x_1, x_2, \dots, x_n be n observations, and let \bar{x} be their arithmetic mean and σ^2 be their variance.

Statement-1 : Variance of $2x_1, 2x_2, \dots, 2x_n$ is $4\sigma^2$.

Statement-2 : Arithmetic mean of

$2x_1, 2x_2, \dots, 2x_n$ is $4\bar{x}$.

- (1) Statement-1 is true, Statement-2 is false.
 (2) Statement-1 is false, Statement-2 is true.
 (3) Statement-1 is true, Statement-2 is true ; Statement-2 is a correct explanation for Statement-1.
 (4) Statement-1 is true, Statement-2 is true ; Statement-2 is not a correct explanation for Statement-1.

Ans. (1)

60. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and

$\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k is equal to:

- (1) 0 (2) -1 (3) $\frac{2}{9}$ (4) $\frac{9}{2}$

Ans. (4)