AIEEE Solved Paper - 2011

Physics

31. 100 g of water is heated from 30ºC to 50ºC. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/Kg/K)
(1) 4.2 kJ (2) 8.4 kJ
(3) 84 kJ (4) 2.1 kJ.

Key. (2)

Sol. $\Delta U = 0.1 \times 4184 \times 20 \cong 8.4 \text{ kJ}$.

$\therefore$ (2).

32. The half life of a radioactive substance is 20 minutes. The approximate time internal ($t_2 - t_1$) between the time t_2 when $\frac{2}{3}$ of it had decayed is
(1) 7 min (2) 14 min
(3) 20 min (4) 28 min.

Key. (3)

Sol. $\dfrac{e^{-\lambda t_1}}{e^{-\lambda t_2}} = 2$

$\Rightarrow t_2 - t_1 = \dfrac{\ell n 2}{\lambda} = T_{\frac{1}{2}} = 20 \text{ min}.$

$\therefore$ (3).

33. A mass M, attached to a horizontal spring, executes SHM with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\dfrac{A_1}{A_2}\right)$ is
(1) $\dfrac{M}{M+m}$ (2) $\dfrac{M+m}{M}$
(3) $\left(\dfrac{M}{M+m}\right)^{1/2}$ (4) $\left(\dfrac{M+m}{M}\right)^{1/2}$.

Key. (4)

Sol. COM $\Rightarrow MA_1\sqrt{\dfrac{k}{M}} = (M+m)V$

Also $V = A_2\sqrt{\dfrac{k}{M+m}}$.

$\therefore$ (4).

34. Energy required for the electron excitation in Li^{++} from the first to the third Bohr orbit is
(1) 12.1 eV (2) 36.3 eV
(3) 108.8 eV (4) 122.4 eV.

Key. (3)

Sol. $\Delta U = 13.6(3)^2\left(\dfrac{1}{1^2} - \dfrac{1}{3^2}\right) = 108.8 \text{ eV}$

$\therefore$ (3).

35. The transverse displacement y (x, t) of a wave on a string is given by
$y(x,t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}\,xt)}$
This represents a
(1) wave moving in +x direction with speed $\sqrt{\dfrac{a}{b}}$
(2) wave moving in +x direction with speed $\sqrt{\dfrac{b}{a}}$
(3) standing wave of frequency $\sqrt{b}$
(4) standing wave of frequency $\dfrac{1}{\sqrt{b}}$.

Key. (2)

Sol. $y(x,\, t) = e^{-(\sqrt{a} \times + \sqrt{b} t)^2}$

$\therefore$ (2).

36. A resistor R and 2μF capacitor in series in connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R make the bulb light up 5 s after the switch has been closed ($\log_{10} 2.5 = 0.4$)

(1) $1.3 \times 10^4\ \Omega$ (2) $1.7 \times 10^5\ \Omega$
(3) $2.7 \times 10^6\ \Omega$ (4) $3.3 \times 10^7\ \Omega$.

Key. (3)

Sol. $V = V_0\left(1 - e^{-\frac{t}{RC}}\right)$

$\Rightarrow 120 = 200\left(1 - e^{\frac{-5}{R\times2\times10^{-6}}}\right)$

$\Rightarrow R = 2.7 \times 10^6\ \Omega$

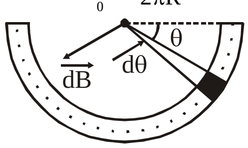
$\therefore$ (3)

37. A current I flows in a infinitely long wire with cross section in the form of a semi–circular ring of radius R. The magnitude of the magnetic induction along its axis is

(1) $\frac{\mu_0 I}{\pi^2 R}$ (2) $\frac{\mu_0 I}{2\pi^2 R}$
(3) $\frac{\mu_0 I}{2\pi R}$ (4) $\frac{\mu_0 I}{4\pi R}$.

Key. (1)

Sol. $B = \int dB \sin\theta = \int_0^{\pi} \frac{\mu_0\left(\frac{I}{\pi}\cdot d\theta\right)}{2\pi R}\sin\theta = \frac{\mu_0 I}{\pi^2 R}$.



$\therefore$ (1).

38. A Carnot engine operating between temperatures T_1 and T_2 has efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively :

(1) 372 K and 310 K
(2) 372 K and 330 K
(3) 330 K and 268 K
(4) 310 K and 248 K.

Key. (1)

Sol. $\eta = 1 - \frac{T_2}{T_1}$

$\therefore$ (1).

39. An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by

$$\frac{dv}{dt} = -2.5\sqrt{v}$$

where v is the instantaneous speed. The time taken by the object, to come to rest, would be

(1) 1 s (2) 2 s
(3) 4 s (4) 8 s.

Key. (2)

Sol. $\int_{6.25}^{0} \frac{d\theta}{\sqrt{v}} = -2.5\int_0^t dt$

$\Rightarrow t = 2$ s

$\therefore$ (2)

40. The electrostatic potential inside a charged spherical ball is given by $\phi = a r^2 + b$ where r is the distance from the centre; a, b are constants. Then the charge density inside the ball is

(1) $-24\pi a\varepsilon_0 r$ (2) $-6 a\varepsilon_0 r$
(3) $-24\pi a\varepsilon_0$ (4) $-6 a\varepsilon_0$.

Key. (4)

Sol. $\phi = ar^2 + b \Rightarrow E = -2ar$

Now, $\oint\!\!\oint_{\text{sphere}} \vec{E}\cdot d\vec{s} = \frac{q_{encl}}{\varepsilon_0}$

$-2ar \cdot 4\pi r^2 = \frac{\rho\cdot\frac{4}{3}\pi r^3}{\varepsilon_0}$

$\Rightarrow \rho = -6a\varepsilon_0$

$\therefore$ (4).

41. A car is fitted with a convex side–view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is

(1) $\frac{1}{10}$ m / s (2) $\frac{1}{15}$ m / s
(3) 10 m/s (D) 15 m/s.

Key. (2)

Sol. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\Rightarrow -\frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$

$\Rightarrow \frac{dv}{dt} = 15\left(\frac{280}{15\times280}\right)^2 \cong \frac{1}{15}$ m / s

$\therefore$ (2).

42. If a wire is stretched to make it 0.1% longer, its resistance will

(1) increase by 0.05%
(2) increase by 0.2%
(3) decrease by 0.2%
(4) decrease by 0.05%.

Key. (2)

Sol. $R = \rho\frac{\ell}{A} = \frac{\rho\ell^2}{(\text{Volume})}$

$\Rightarrow R \propto \ell^2$

$\therefore \quad \frac{\Delta R}{R} = 2\frac{\Delta \ell}{\ell}$

$\therefore$ (2).

43. Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is

(1) $\frac{(T_1 + T_2 + T_3)}{3}$

(2) $\frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$

(3) $\frac{n_1T_1^2 + n_2T_2^2 + n_3T_3^3}{n_1T_1 + n_2T_2 + n_3T_3}$

(4) $\frac{n_1^2T_1^2 + n_2^2T_2^2 + n_3T_3^3}{n_1T_1 + n_2T_2 + n_3T_3}$.

Key. (2)

Sol. Number of moles of first gas $= \frac{n_1}{N_A}$

Number of moles of second gas $= \frac{n_2}{N_A}$

Number of moles of thirst gas $= \frac{n_3}{N_A}$

If no loss of energy then

$P_1V_1 + P_2V_2 + P_3V_3 = PV$

$\frac{n_1}{N_A}RT_1 + \frac{n_2}{N_A}RT_2 + \frac{n_3}{N_A}RT_3$

$= \frac{n_1 + n_2 + n_3}{N_A}RT_{mix}$

$T_{mix} = \frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$.

$\therefore$ (2).

44. Two identical charged spheres suspended from a common point by two massless strings of length ℓ are initially a distance d ($d << \ell$) apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between them,

(1) $v \propto x^{\frac{1}{2}}$ (2) $v \propto x^{-1}$

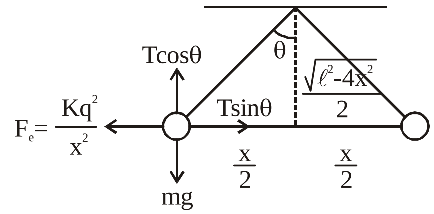
(3) $v \propto x^{-\frac{1}{2}}$ (4) $v \propto x$.

Key. (3)

Sol. $T\sin\theta = \frac{Kq^2}{x^2}$ …(i)

$T\cos\theta = mg$ …(ii)

$\text{Tan}\theta = \frac{x}{\sqrt{\ell^2 - 4x^2}} = \frac{x}{\ell}$



$\frac{x}{\ell} = \frac{kq^2}{x^2mg}$

$x^3 = \frac{kq^2\ell}{mg}$ …(i)

$x^3 \propto q^2$

$3x^2\frac{dx}{dt} \propto 2q\frac{dq}{dt}$

$x^2 \cdot v \propto q$

$v \propto x^{-\frac{1}{2}}$.

$\therefore$ (3)

45. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (surface tension of soap solution = 0.03 Nm^{-1})

(1) 4 π mJ (2) 0.2 π mJ

(3) 2 π mJ (4) 0.4 π mJ.

Key. (4)

Sol. W = (surface energy)$_{final}$ – (surface energy)$_{initial}$

$W = T \times 4\pi\left[\left(5 \times 10^{-4}\right) (3 \times 10^{-2}\right] \times 2 \times$

$= 4\pi \times 0.03 \times 16 \times 10^{-4} \times 2$

$= 4\pi \times 0.48 \times 10^{-4} \times 2$

$= 1.92\pi \times 10^{-4} \times 2$

$= 3.94\pi \times 10^{-4} = 0.394\ \pi\ mJ \approx 0.4\pi\ mJ$.

$\therefore$ (4).

46. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at t = 0. The time at which the energy is stored equally between the electric and the magnetic fields is

(1) $\pi\sqrt{LC}$ (2) $\frac{\pi}{4}\sqrt{LC}$

(3) $2\pi\sqrt{LC}$ (4) $\sqrt{LC}$.

Key. (2)

Sol. $\frac{q_0^2}{2C} = \frac{q^2}{2C} + \frac{Li^2}{2}$

differentiating w.r.t. t

$\frac{di}{dt} = -\frac{q}{LC}$

$\frac{d^2q}{dt^2} = -\frac{1}{LC}q$

Comparing $\frac{d^2x}{dt^2} = -\ \omega^2 x$

$\omega = \frac{1}{\sqrt{LC}}$

So, $q = q_0 \cos\omega t$ ($\because$ at t = 0, $q = q_0$)

For half energy $q = \frac{q_0}{\sqrt{2}}$

So, $\frac{q_0}{\sqrt{2}} = q_0 \cos\omega t$

$\omega t = \frac{\pi}{4}$

$t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{LC}$.

$\therefore$ (2).

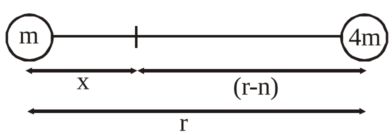
47. Two bodies of masses m and 4 m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is

(1) zero (2) $-\frac{4Gm}{r}$

(3) $-\frac{6Gm}{r}$ (4) $-\frac{9Gm}{r}$.

Key. (4)

Sol.



Let gravitational field at P is zero

$\frac{Gm}{x^2} = \frac{G \times 4m}{(r = n)^2}$

$x = \frac{r}{4}$

Now potential at P

$V_P - = \frac{Gm}{x}\ \ \frac{G(4m)}{(r-n)}$

$= -\frac{Gm}{(r/3)}\ \ \frac{4Gm}{(2r/3)}$

$= -\frac{9Gm}{r}$.

$\therefore$ (4).

48. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc of reach its other end. During the journey of the insect, the angular speed of the disc

(1) remains unchanged

(2) continuously decreases

(3) continuously increases

(4) first increases and then decreases.

Key. (4)

Sol. Angular momentum $L = I\omega$

$L = mr^2.\omega$

Since r first decrease then increases

So due to conservation of angular momentum L first increases then decreases.

49. Let the x – z plane be the boundary between two transparent media. Medium 1 in $z \ge 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j}\ \ 10\hat{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is

(1) 30º (2) 45º

(3) 60º (4) 75º.

Key. (2)

Sol. Angle of incidence with Z direction (normal)

$\cos\alpha = \frac{10}{\sqrt{\left(6\sqrt{3}\right)^2 + \left(8\sqrt{3}\right)^2 + (10)^2}} = \frac{1}{2}$

$\alpha = 60º$.

So, $\mu_1 \sin\alpha = \mu_2 \sin\beta$

$\sqrt{2} \times \sin 60 = \sqrt{3}\sin\beta$

$\beta = 45º$.

$\therefore$ (2)

50. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x–axis. Their mean position is separated by distance X_0 ($X_0 > A$). If the maximum separation between them is (X_0 + A), the phase difference between their motion is

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$.

Key. (2)

51. **Direction :**
The question has a paragraph followed by two statements, **Statement – 1** and **Statement – 2**. Of the given four alternatives after the statements, choose the one that describes the statements.

A thin air film is formed by putting the convex surface of a plane–convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

Statement – 1 :
When light reflects from the air–glass plate interface, the reflected wave surfers a phase change of π.

Statement – 2 :
The centre of the interference pattern is dark.

(1) Statement – 1 is True, Statement – 2 is False.
(2) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.
(3) Statement – 1 is True, Statement – 2 is True; Statement – 2 is not the correct explanation for Statement – 1.
(4) Statement – 1 is False, Statement – 2 is True.

Key. (2)

52. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by

(1) $\frac{(\gamma-1)}{2(\gamma+2)R}Mv^2K$
(2) $\frac{(\gamma-1)}{2\gamma R}Mv^2K$
(3) $\frac{\gamma Mv^2}{2R}K$
(4) $\frac{(\gamma-1)}{2R}Mv^2K$.

Key. (4)

Sol. $\frac{1}{2}Mv^2 = \frac{R}{\gamma-1}\Delta T$

$\Rightarrow \Delta T = \frac{(\gamma-1)}{2R}Mv^2\,K$.

$\therefore$ (4)

53. A screw gauge gives the following reading when used to measure the diameter of a wire.
Main scale reading : 9 mm
Circular scale reading : 52 divisions
Given that 1 mm on main scale corresponds to 100 divisions of the circular scale.
The diameter of wire from the above data is

(1) 0.52 cm (2) 0.052 cm
(3) 0.026 cm (4) 0.005 cm.

Key. (2)

Sol. d = MSR + CSR

$= 0 + 52 \times \frac{1}{100} = 0.52$ mm.

$\therefore$ (2)

54. A boat is moving due east in a region where the earth's magnetic field is 5.0×10^{-5} NA^{-1} m^{-1} due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the induced emf in the wire of aerial is

(1) 1 mV (2) 0.75 mV
(3) 0.50 mV (4) 0.15 mV.

Key. (4)

Sol. $\varepsilon_{ind} = Bv\ell$

$= 5\times10^{-5} \times 1.50 \times 2 = 0.15$ mV.

$\therefore$ (4)

55. **Direction :**
The question has **Statement – 1** and **Statement – 2**. Of the four choices given after the statements, choose the one that describes the two statements.

Statement – 1 :
Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.

Statement – 2 :
The state of ionosphere varies from hour to hour, day to day and season to season.

(1) Statement – 1 is True, Statement – 2 is False.
(2) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.
(3) Statement – 1 is True, Statement – 2 is True; Statement – 2 is not the correct explanation for Statement – 1.
(4) Statement – 1 is False, Statement – 2 is True.

Key. (2)

Sol.

56. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is

(1) $\frac{3}{2}g$ (2) g

(3) $\frac{2}{3}g$ (4) $\frac{g}{3}$.

Key. (3)

Sol. Equations of motion are

$mg - T = ma$ …(i)

and $T \cdot R = \frac{1}{2}mR^2\alpha$ …(ii)

and $a = R\alpha$ …(iii)

Solving $a = \frac{2}{3}g$.

∴ (3)

57. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is

(1) $\pi\frac{v^2}{g}$ (2) $\pi\frac{v^4}{g^2}$

(3) $\frac{\pi}{2}\frac{v^4}{g^2}$ (4) $\pi\frac{v^2}{g^2}$.

Key. (2)

Sol. $A = \pi R_{max}^2 = \frac{\pi v^4}{g^2}$.

∴ (2)

58. **Direction :**

The question has **Statement – 1** and **Statement – 2**. Of the four choices given after the statements, choose the one that describes the two statements.

Statement – 1 :

A metallic surface is irradiated by a monochromatic light of frequency $v > v_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.

Statement – 2 :

The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.

(1) Statement – 1 is True, Statement – 2 is False.

(2) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.

(3) Statement – 1 is True, Statement – 2 is True; Statement – 2 is not the correct explanation for Statement – 1.

(4) Statement – 1 is False, Statement – 2 is True.

Key. (4)

Sol. $K_{max} = hv - w$

and $K_{max} = eV_S$.

∴ (4)

59. A pulley of radius 2 m is rotated about its axis by a force F = (20t – 5t^2) Newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations make by the pulley before its direction of motion if reversed, is

(1) less than 3

(2) more than 3 but less than 6

(3) more than 6 but less than 9

(4) more than 9.

Key. (2)

Sol. $\alpha = \frac{\tau}{I} = 4t - t^2$

$\Rightarrow \frac{d\omega}{dt} = 4t - t^2$

$\Rightarrow \omega = 2t^2 - \frac{t^3}{3}$

ω is zero at t = 0s and t = 6s

Now $\frac{d\theta}{dt} = \omega = 2t^2 - \frac{t^3}{3}$

$\Rightarrow \theta = \frac{2}{3}t^3 - \frac{t^4}{12}$

θ at t = 6s = 36 rad

∴ number of rotations $= \frac{36}{2\pi} < 6$.

∴ (2).

60. Water is flowing continuously from a tap having an internal diameter 8 × 10^{-3} m. the water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2 × 10^{-1} m below the tap is close to

(1) 5.0 × 10^{-3} m (2) 7.5 × 10^{-3} m

(3) 9.6 × 10^{-3} m (4) 3.6 × 10^{-3} m.

Key. (4)

Sol. $A_1v_1 = A_2v_2$

and $v_2^2 = v_1^2 + 2gh$.

∴ (4).