

PART – A (ONE MARKS QUESTION)

- (1) Show that $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$.
- (2) Find the maximum and minimum values, if any of $f(x) = |\sin 3x| - 3$.
- (3) Find the direction cosines of x-axis. **Ans 1, 0, 0**
- (4) Find the value of $\cos[\sec^{-1} x + \operatorname{cosec}^{-1} x]$.
- (5) Let $f, g : R \rightarrow R$ be defined as $f(x) = |x|$ and $g(x) = [x]$ where $[x]$ denotes the greatest integer less than or equal to x . Find $g \circ f(-\sqrt{2})$. **Ans 1**
- (6) Find x if $\tan^{-1} 4 + \cot^{-1} x = \frac{\pi}{2}$.
- (7) If $\vec{AB} = 4\hat{i} + 5\hat{j} - 3\hat{k}$ and the position vector of point B is $3\hat{i} - \hat{j} + 2\hat{k}$, find the position vector of point A. **Ans $-i - 6j + 5k$**
- (8) Evaluate: $\int_0^a \frac{1}{x + \sqrt{a^2 - x^2}} dx$.
- (9) Let $f : R - \{-\frac{3}{5}\} \rightarrow R$ be a function as $f(x) = \frac{2x}{5x+3}$, Find f^{-1} .
- (10) Find the value of k the system of equation has non-trivial solution $x - 2y + 3z = 0$; $3x - y + 2z = 0$ & $2x + ky + z = 0$ **$-i - 6j + 5k$** .
- (11) Find the value of $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$.
- (12) If $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric then find the value of x .
- (13) Determine the integrating factor of differential equations: $(x - \sin y)dy + \tan y dx = 0, y(0) = 0$.
- (14) In a triangle ABC, the sides AB and BC are represented by vectors $2\hat{i} - \hat{j} + 2\hat{k}$, $\hat{i} + 3\hat{j} + 5\hat{k}$ respectively. Find the vector representing CA. **$-3i - 2j - 7k$**
- (15) Write the value of λ such that the line $\frac{x-2}{9} = \frac{y-1}{\lambda} = \frac{z+3}{-6}$ is perpendicular to the plane $3x - y - 2z = 7$. **-3**
- (16) Find the equations of planes parallel to the plane $x - 2y + 2z = 3$ which are at a unit distance from the point $(1, 2, 3)$.
- (17) If the following matrix is skew symmetric, find the values of a, b, c . $A = \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$.
- (18) Evaluate: $\int (e^x \log a + e^a \log x + e^a \log a) dx$.

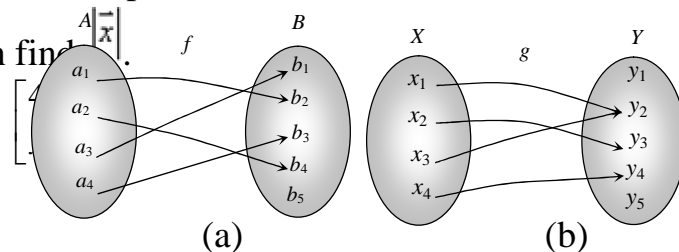
(19) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & K & 1 \end{bmatrix}$, Find k , If Cofactor of a_{11} is twice the cofactor of a_{23} .

(20) Find the perpendicular distance from $(2, 5, 6)$ on XY plane.

(21) If \vec{a} is a unit vector and $(\vec{x} + \vec{a}) \cdot (\vec{x} - \vec{a}) = 8$ then find $|\vec{x}|$.

(22) For what value of x , the matrix is singular?

(23) Write the name of function.



(24) If the graph of $y = f(x)$ is given and the line parallel to x -axis cuts the curve at more than one point. Write the name of function.

(25) Check the monotonicity i.e increasing & decreasing of $f(x) = \cos 2x, [\pi/2, \pi]$.

(26) Let $\vec{a} = 5\vec{i} - \vec{j} + 7\vec{k}$, $\vec{b} = \vec{i} - \vec{j} + \lambda\vec{k}$ Find λ such that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular.

(27) Find $(\vec{i} \times \vec{j}) \cdot \vec{k} + (\vec{k} \times \vec{j}) \cdot \vec{i} - (\vec{i} \times \vec{k}) \cdot \vec{j}$.

(28) The probability that an event happens in one trial of an experiment is .4 . Three independent trials of the experiment are performed . Find the probability that the event happens at least once .

(29) Evaluate : $\int [1 + 2 \tan x(\tan x + \sec x)]^{1/2} dx$.

(30) Find the value of a and b such that the function f defined by $f(x) = \begin{cases} 5 & \text{if } x \leq 2 \\ ax + b & \text{if } 2 < x < 10 \\ 21 & \text{if } x \geq 10 \end{cases}$.

(31) If $|\vec{a} \times \vec{b}| = 4$, $|\vec{a} \cdot \vec{b}| = 2$, then find $|\vec{a}|^2 |\vec{b}|^2$.

(32) If $f(x) = |x|$ and $g(x) = |5x - 2|$ find $f \circ g$ and $g \circ f$.

(33) Find the value of $\tan^{-1}(-1) + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$.

(34) Show that $*$: $\mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ given by $(a, b) \rightarrow a + 4b^2$ is a binary operation.

(35) Determine the order and degree of the differential equation $\left(\frac{d^2x}{dx^2}\right) = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$.

(36) Find the projection $\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$, on $\vec{b} = \vec{i} - 2\vec{j} + \vec{k}$.

(37) Find the angle made by the vector $\vec{i} - 4\vec{j} + 8\vec{k}$ with the z -axis.

(38) Evaluate : $\int \frac{dx}{x^2(x^4 + 1)^{3/4}}$.

(39) Find the inverse element of the binary relation $a \otimes b = a + b - 4$.

- (40) Evaluate $\int \frac{(x+1)(x+\log x)^2}{x} dx$.
- (41) Find the number of binary operations on the set $A = \{a, b\}$.
- (42) If $f(x) = x^2 - 1$ and $g(x) = 2x + 3$ find $f \circ g(2)$.
- (43) Given $P(A) = 1/2$, $P(B) = 1/3$ and $P(A \cup B) = 2/3$. Are the events A and B independent?
- (44) If $|A| = 3$ find the $|A^{-1}|$.
- (45) Find $\int_{-\pi}^{\pi} (\sin^{-93} x + x^{295}) dx$.
- (46) Find values of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$.
- (47) Solve for x : $\tan^{-1} \left(\frac{1-x}{1+x} \right) = \frac{1}{2} \tan^{-1} x$; $x > 0$.
- (48) State whether the function $f : N \rightarrow N$ given by $f(x) = 5x$ is injective, surjective or both.
- (49) Solve for x : $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$.
- (50) Find $|\vec{a} - \vec{b}|$ if two vectors \vec{a} and \vec{b} are such that $|\vec{a}| = 2$; $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$.
- (51) Let $*$ be the binary operation on N given by $a*b = \text{LCM of } a \text{ and } b$. Find the value of $20*16$. Is $*$ (i) commutative, (ii) associative?
- (52) Find the angle between the lines $\frac{x+1}{5} = \frac{y-2}{-2} = \frac{z-1}{2}$ & $\frac{x+3}{-2} = \frac{z-4}{3}$, $y = -5$.
- (53) Let $*$ be a binary operation on the set Q of rational numbers given as $a*b = (2a-b)^2$, $a, b \in Q$. Find $3*5$ and $5*3$?
- (54) Solve the differential equations: $\sqrt{1+x^2} dy + \sqrt{1+y^2} dx = 0$.
- (55) A line makes an angle of $\frac{\pi}{4}$ with each x-axis and y-axis. Find the angle between this line and the z-axis.
- (56) Find the value of k for which the matrix $\begin{pmatrix} k & 2 \\ 3 & 4 \end{pmatrix}$ has no inverse.
- (57) Find the area of the parallelogram whose adjacent sides are the vectors $\hat{i} - \hat{j} + 3\hat{k}$ and $2\hat{i} - 7\hat{j} + \hat{k}$.
- (58) The probability of a man hitting a target is $\frac{1}{3}$, if he fires 3 times, what is the probability of his hitting at least once?
- (59) Find the value of x, y and z so that the vectors $\vec{a} = 2x\hat{i} + 3\hat{j} + z\hat{k}$ and $\vec{b} = 2\hat{i} + y\hat{j} + z\hat{k}$ are equal.

(60) Let \vec{a} and \vec{b} be two vectors such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector. Then

what is the angle between \vec{a} and \vec{b} ?

(61) If $\begin{bmatrix} x+2y & -y \\ 3x & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 6 & 4 \end{bmatrix}$, find the value of x and y.

(62) If $\begin{vmatrix} x & x \\ 1 & x \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$, find the value of x.

(63) Given an example to show that the relation R in the set of natural numbers defined by $R = \{ (x, y) : x, y \in \mathbb{N}, x \leq y^2 \}$ is not transitive.

(64) If $\int_0^a 3x^2 dx = 8$, find the value of a.

(65) Find f(x) satisfying the following : $\int (2x+1)\sqrt{x^2+x+1} dx = f(x) + c$.

(66) Evaluate : $\int_0^{2/3} \frac{dx}{4+9x^2}$.

(67) Determine the binomial distribution whose mean is 9 and whose standard deviation is 3/2.

(68) Find the value of x if the area of Δ is 35 square cms with vertices (x, 4), (2, -6) and (5, 4).

(69) The slope of the curve $2y^2 = ax^2 + b$ at (1, -1) is -1. Find a and b.

(70) Find the value of x such that: $\begin{pmatrix} x & -5 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} \begin{pmatrix} x \\ 4 \\ 1 \end{pmatrix} = 0$.

(71) If $P(A)=0.5$, $P(B)=0.6$ and $P(A \cup B)=0.8$, find $P(A/B)$.

(72) Determine the k so that the function $f(x) = \begin{cases} \frac{\sin 2x}{5x} & ; \text{if } x \neq 0 \\ k & ; \text{if } x = 0 \end{cases}$, is continuous at $x=0$.

(73) Find $\frac{dy}{dx}$ if $x^5 + y^5 + 5xy = 100$.

(74) Evaluate : $\int_0^{2.5} [x^2] dx$, where [] stands for greatest integer function.

(75) Find the derivative of $\tan x$ w.r.t. $\sin x$.

(76) Let $f : R \rightarrow R$ be defined by $f(x) = x^2 - 3x + 1$, Find $f[f(x)]$.

(77) Find whether the relation R in the set $A = \{1, 2, 3\}$ given by $R = \{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$ is transitive.

(78) Find the condition that the matrices A and B will be inverse of each other.

(79) If $\sin^{-1} x = y$, then write the range of y.

- (80) Evaluate $\int_0^{\pi/2} \frac{\sin x dx}{1 + \cos^2 x}$.
- (81) The value of $\tan\left(\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right)$.
- (82) Show that the relation R on $N \times N$ defined by $(a,b)R(c,d)$ if and only if $ad = bc \forall a,b,c,d \in N$ is transitive.
- (83) Find the condition that the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ may be invertible .
- (84) At what point on the curve $x^2 + y^2 - 4y - 5 = 0$, the tangent to the curve is parallel to the y-axis ?
- (85) Evaluate $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{1}{2}\right)\right]$.
- (86) For the curve $y = 3x^2 + 4x$, find the slope of the tangent to the curve at the point where it cuts the x axis .
- (87) Find whether $f(x) = \frac{1}{x}$ is strictly increasing or strictly decreasing function.
- (88) If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$, then find the values of x .
- (89) If the given lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, find k .
- (90) Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$ Find a matrix D such that $CD - AB = O$.
- (91) Find the value of $\cos[\sec^{-1}x + \cos^{-1}x]$.
- (92) Determine the integrating factor of differential equations: $(x - \sin y)dy + \tan y dx = 0$, $y(0) = 0$.
- (93) The odds against A solving a certain problem are 4 to 3 and the odds in favour of B solving the same problem are 7 to 5 . Find the probability that the problem will be solved.'
- (94) If $2\cos^{-1}\sqrt{\frac{1+x}{2}} = \frac{\pi}{2}$, then find x .
- (95) Let * be a binary operation on R. If $a * b = a + b + ab$; $a, b \in R$. Find x such that $(2 * x) * 3 = 7$.
- (96) Let * be a binary operation on N. If $a * b = \text{lcm of } a \text{ \& } b$; $a, b \in N$. Find $(2 * 4) * 6$.
- (97) Find the equations of planes parallel to the plane $x - 2y + 2z = 3$ which are at a unit distance from the point (1, 2, 3).
- (98) If $4\sin^{-1}x + \cos^{-1}x = \pi$ then find the value of x .
- (99) There are three mutually exclusive and exhaustive events E_1, E_2 and E_3 . The odds are 8:3 against E_1 and 2:5 in favor of E_2 . Find the odds against E_3 .
- (100) Prove that : $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$.

- (101) Evaluate: $\int \frac{e^{5 \log x} - e^{4 \log x}}{e^{3 \log x} - e^{2 \log x}} dx$.
- (102) Write the equation of the plane containing the lines $\vec{r} = \vec{a} + \lambda \vec{b}$ & $\vec{r} = \vec{a} + \mu \vec{c}$.
- (103) Write the value of k for which the line $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-1}{k}$ is perpendicular to the normal to the plane $\vec{r} \cdot (2i + 3j + 4k) + 4 = 0$.
- (104) At what points of the ellipse $16x^2 + 9y^2 = 400$, does the ordinates decrease at the same rate at which the abscissa increase?
- (105) Find the inverse element of the binary relation $a \otimes b = a + b - 4$.
- (106) Given $\vec{a} \cdot \vec{b} = \left| \vec{a} \times \vec{b} \right|$ find the angle between \vec{a} & \vec{b} .
- (107) Write a vector normal to the plane $\vec{r} = \mu \vec{a} + \lambda \vec{b}$.
- (108) If $A = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$ find matrix B such that $AB = I$.
- (109) Write the equation of plane passing through (1,2,3) and perpendicular to line $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{-1}$ in vector form.
- (110) The slope of tangent to curve $y = \frac{x-1}{x-2} \text{ at } x = 10$.
- (111) Write a equation of the plane $\vec{r} = \vec{a} + \lambda \vec{b} + \mu \vec{c}$.
- (112) Write a general equation of the plane parallel to x axis.
- (113) If $A^2 = A$ for $A = \begin{bmatrix} -1 & b \\ -b & 2 \end{bmatrix}$, then find the value of b.
- (114) If A and B are two events such that $P(A) = 0.3, P(B) = 0.6$ & $P(B/A) = 0.5$, find $P(A \cup B)$.
- (115) Find the values of a and b so that the function f given by $f(x) = \begin{cases} 1, & \text{if } x \leq 3 \\ ax + b, & \text{if } 3 < x < 5 \\ 7, & \text{if } x \geq 5 \end{cases}$.
- (116) Find a 2×2 matrix B such that $B \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$.
- (117) If $A = \begin{bmatrix} 2 & 4 & -1 \\ 2 & -3 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, Find the value of $A(\text{Adj}.A)$.
- (118) Evaluate : $\sec^2(\tan^{-1}2) + \text{cosec}^2(\cot^{-1}3)$.
- (119) Show that $y = ae^{2x} + be^{-x}$ is a solution of the differential equation $\frac{d^2 y}{dx^2} - \frac{dy}{dx} - 2y = 0$.

(120) If $X = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$. Find $|3X|$. **Ans $3^3|x|$**

(121) If \vec{a} & \vec{b} are two unit vectors such that $|\vec{a} + \vec{b}| = \sqrt{3}$, find $(2\vec{a} - 5\vec{b})(3\vec{a} + \vec{b})$.

(122) If the points (3, -2), (x, 2), (8, 8) are collinear, find x using determinant.

(123) Find the points on the curve $y = x^3 - 2x^2 - x$ at which the tangent lines are parallel to the line $y = 3x - 2$.

(124) If $\vec{a} = \hat{i} + \hat{j}$; $\vec{b} = \hat{j} + \hat{k}$; $\vec{c} = \hat{k} + \hat{i}$; find a unit vector in the direction of $\vec{a} + \vec{b} + \vec{c}$.

(125) Prove that $f(x) = \frac{e^{1/x} - 1}{e^{1/x} + 1}$, does not exist at $x = 0$.

(126) If $[1 - 1x] \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$, find x.

(127) Evaluate: $\int_{-\pi/2}^{\pi/2} \log\left(\frac{2 - \sin x}{2 + \sin x}\right) dx$.

(128) If $|\vec{a}| = \sqrt{26}$, $|\vec{b}| = 7$ & $|\vec{a} \times \vec{b}| = 35$, find $\vec{a} \cdot \vec{b}$.

(129) For the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, find M_{12} and C_{23} where M_{12} is minor of the element in first row and second column and C_{23} is cofactor of the element in second row and third column.

(130) If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then find the value of x.

(131) Find λ so that the four points with p.v. $-\hat{j} + \hat{k}$, $2\hat{i} - \hat{j} - \hat{k}$, $\hat{i} + \lambda\hat{j} + \hat{k}$ and $3\hat{j} + 3\hat{k}$ are coplanar.

(132) Find the magnitude of two vectors \vec{a} and \vec{b} , having the same magnitude and such that the angle between them is 60° and their scalar product is $\frac{1}{2}$.

(133) Determine degree of differential equations (i) $\left(\frac{d^2y}{dx^2}\right)^2 + \frac{dy}{dx} = y$ (ii) $\frac{d^4y}{dx^4} + 2x\left(\frac{dy}{dx}\right)^3 = 0$.

(134) If A, B, C are three non zero square matrix of same order, find the condition on A such that $AB = AC \Rightarrow B = C$. **Ans $|A| \neq 0$**

(135) If f(x) is real function, then find $\int_0^{2a} \frac{f(x)}{f(x) + f(2a-x)} dx$.

(136) Let $f : R \rightarrow R$ be defined by $f(x) = x^2 - 3x + 1$, Find $f[f(x)]$.

(137) Find x, y if the points (x, -1, 3), (3, y, 1) and (-1, 11, 9) are collinear.

- (138) Let f, g be the function $f = \{(1, 5), (2, 6), (3, 4)\}$, $g = \{(4, 7), (5, 8), (6, 9)\}$. What is the range of f and g ?
- (139) If $f(x) = \frac{x-1}{x+1}$ ($x \neq 1, -1$), show that $f \circ f^{-1}$ is an identity function.
- (140) Find values of k if area of triangle is 4 square units and vertices are $(k,0), (4,0), (0,2)$.
- (141) The number of all possible matrices of order 3×3 with each entry 0 or 1 .
- (142) Find the values of λ for which the homogeneous system of equations:

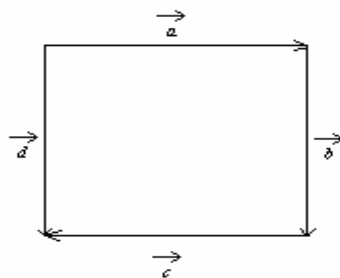
$$\begin{aligned} 2x + 3y - 2z &= 0 \\ 2x - y + 3z &= 0 \\ 7x + \lambda y - z &= 0 \end{aligned}$$
 find non-trivial solutions.
- (143) If the probability that a man aged 60 will live to be 70 is 0.4, what is the prob. that out of 3 men now 60, at least 2 will live to be 70 ?
- (144) Find the coordinates of the point on the curve $y = x^2 - 6x + 9$ where the normal is parallel to the line $y = x + 5$.
- (145) A five digit number is formed by the digits 1, 2, 3, 4, 5 without repetition. Find the probability that the number is divisible by 4 .
- (146) Evaluate: $\tan\left[2 \tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right]$.
- (147) Let $f : R \rightarrow R$ be defined by $f(x) = x^2 + 1$, Find $f^{-1}(26)$.
- (148) If $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, Find x and y .
- (149) If matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \frac{1}{K} \text{adj}(A)$, then find the value of K .
- (150) Write the principal value of $\cos^{-1}\left(\cos \frac{5\pi}{3}\right)$
- (151) Find x , if $\begin{pmatrix} 5 & 3x \\ 2y & z \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 12 & 6 \end{pmatrix}^t$
- (152) For what value of a , $\begin{pmatrix} 2a & -1 \\ -8 & 3 \end{pmatrix}$ is a singular matrix?
- (153) A square matrix A , of order 3, has $|A| = 5$, find $|A \cdot \text{adj}A|$.
- (154) Evaluate $\int 5^x dx$
- (155) Write the value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^5 x dx$.
- (156) Find the position vector of the midpoint of the line segment joining the points $A(5\hat{i} + 3\hat{j})$ and $B(3\hat{i} - \hat{j})$
- (157) If $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = (6\hat{i} + \lambda\hat{j} + 9\hat{k})$ and $\vec{a} \parallel \vec{b}$, find the value of λ

- (158) Find the distance of the point (a, b, c) from x-axis.
- (159) Give an example of two non zero 2×2 matrix A,B such that $AB = 0$.
- (160) If $\begin{vmatrix} 1 & 2 & 3 \\ 2 & x & 3 \\ 3 & 4 & 5 \end{vmatrix} = 0$, Find the value of x.
- (161) If a binary operation \oplus is defined by $a \oplus b = 2a - 3b$, find $8 \oplus 3$.
- (162) Write the number of all one-one functions from the set $A = \{a, b, c\}$ to itself.
- (163) What is the value of $|3I_3|$, where I_3 is the identity matrix of order 3 ?
- (164) For what value of k, the matrix $\begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}$ is not invertible?
- (165) If A is a matrix of order 2×3 and B is a matrix of order 3×5 , what is the order of matrix $(AB)'$ or T ?
- (166) Write a value of $\int \frac{dx}{\sqrt{4-x^2}}$.
- (167) Find f(x) satisfying the following : $\int e^x(\sec^2 x + \tan x)dx = e^x f(x) + c$.
- (168) Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar.
- (169) Find the value of λ for which the vector $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ are perpendicular to each other.
- (170) Write the value of λ such that the line $\frac{x-2}{9} = \frac{y-1}{\lambda} = \frac{z+3}{-6}$ is perpendicular to the plane $3x-y-2z = 7$.
- (171) If $\int (e^{ax} + bx)dx = \frac{e^{4x}}{4} + \frac{3x^2}{2}$, find the values of a and b.
- (172) Let * be a binary operation on Z. Find $2*(-4)$ if $a*b=4ab$; $a, b \in Z$.
- (173) Two matrices A and B are given by $A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$ and $B = \frac{4}{5} \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$ such that $AB = I = BA$.

Write the inverse of matrix A.

- (174) The length x of a rectangular is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/min. When $x = 8$ cm and $y = 6$ cm, find the rate of change of the perimeter of rectangle .
- (175) Find the value of p for which the vectors $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are parallel.
- (176) Evaluate $\int_0^{1.5} [x] dx$.
- (177) Evaluate : $\int \frac{3x^2 + 4x - 5}{(x^3 + 2x^2 - 5x + 1)^2} dx$. **Ans $\int_0^1 0 dx + \int_1^{1.5} 1 dx$**

- (178) Evaluate: $\int \frac{1 + \cot x}{x + \log \sin x} dx$
- (179) Find a, for which $f(x) = a(x + \sin x)$ is increasing. **Ans $a \geq 0$**
- (180) If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ such that $\vec{a} + \lambda\vec{b}$ is perpendicular to \vec{c} , then find the value of λ .
- (181) The radius of a circular plate increases at the rate of 0.1 cm/sec. At what rate does the area increase when the radius of the plate is 25 cm?
- (182) There are three mutually exclusive and exhaustive events E_1 , E_2 and E_3 . The odds are 8:3 against E_1 and 2:5 in favor of E_2 . Find the odds against E_3 .
- (183) Let $f: R \rightarrow R$ be defined by $f(x) = x^2 + 5x + 9$, Find $f^{-1}(9)$.
- (184) Evaluate: $\cos \left[\cos^{-1} \left(\frac{-\sqrt{3}}{2} \right) + \frac{\pi}{6} \right]$.
- (185) Find λ when the projection of $\hat{i} + \lambda\hat{j} + \hat{k}$ on $\hat{i} + \hat{j}$ is $\sqrt{2}$ units.
- (186) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$, then find the value of $|\text{adj } A|$.
- (187) If $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ & $\vec{b} = \hat{i} - 3\hat{k}$, find $|\vec{b} \times 2\vec{a}|$.
- (188) Evaluate: $\int_0^{\pi/2} \log \left[\frac{3 + 5 \cos x}{3 + 5 \sin x} \right] dx$.
- (189) If the graph of $y = f(x)$ is given and the line parallel to x -axis cuts the curve at more than one point. Write the name of function.
- (190) Let $A = \{2, 3, 4, 5, 6, 7, 8, 9\}$. Let R be the relation on A defined by $R = \{(x, y) : x \in A, y \in A \text{ and } x \text{ divides } y\}$. Find set R as a order pair.
- (191) Find the value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$.
- (192) The vectors $\vec{a} = 3\hat{i} + x\hat{j} - \hat{k}$ & $\vec{b} = 2\hat{i} + \hat{j} + y\hat{k}$ are mutually perpendicular. Given that $|\vec{a}| = |\vec{b}|$, find the values of x and y .
- (193) If a line makes angle 90° , 60° and 30° with the positive direction x , y and z respectively, find its direction cosines.
- (194) In figure (a square), identify the following vectors. (i) Coinitial (ii) Equal (iii)



Collinear but not equal

- (195) If A is a matrix of order $m \times n$ and C is a column of A , find order of R as a matrix.
- (196) Find a matrix X such that $2A + B + X = 0$, where $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$; $B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$.
- (197) Find the slope of the tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$ at the point $(2, -1)$.
- (198) If $f(1) = 4$; $f'(1) = 2$, find the value of derivative of $\log f(e^x)$ w.r.t. x at the point $x = 0$.
- (199) What is the principal value of: $\sin^{-1}\left(\sin \frac{11\pi}{3}\right) + \cos^{-1}\left(\cos \frac{10\pi}{3}\right)$.
- (200) Given $|\vec{a}| = 10, |\vec{b}| = 2$ & $\vec{a} \cdot \vec{b} = 12$, find $|\vec{a} \times \vec{b}|$.
- (201) Consider the set $A = \{ a, b, c \}$ give an example of a relation R on A which is Symmetric and reflexive but not transitive.
- (202) The matrix $\begin{pmatrix} 1 & a & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{pmatrix}$ is not invertible, then find the value of ' a '.
- (203) Find λ when the scalar projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units.
- (204) Evaluate: $\int 2^{2^x} 2^{2^x} 2^x dx$.
- (205) A random variable X has the following probability distribution: Find a .
- | | | | | | | | | |
|-----------|-----|------|------|------|------|-------|------|------|
| $X:$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $P(X=r):$ | a | $4a$ | $3a$ | $7a$ | $8a$ | $10a$ | $6a$ | $9a$ |
- (206) There are 4 letters and 4 addressed envelopes. Find the probability that all the letters are not dispatched in the right envelopes.
- (207) Evaluate: $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$.
- (208) If A and B are square matrices of order 3 such that $|A| = -1, |B| = 3$, then find the value $|3AB|$.
- (209) Find sine of the angle between the vectors \vec{a} and \vec{b} , if $\vec{a} \times \vec{b} = 3\hat{i} - 6\hat{j} - 5\hat{k}$, $|\vec{a}| = \sqrt{5}$ and $|\vec{b}| = \sqrt{14}$. **Ans $3^3 |A||B|$**
- (210) Write the order and degree of the differential equation $y = x \frac{dy}{dx} + a \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$.
- (211) If \vec{r} is any vector in space, show that $\vec{r} = (\vec{r} \cdot \hat{i})\hat{i} + (\vec{r} \cdot \hat{j})\hat{j} + (\vec{r} \cdot \hat{k})\hat{k}$.
- (212) Evaluate $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$.
- (213) A four digit number is formed using the digit 1,2,3,5 with no repetitions. Find the probability that the number is divisible by 5.
- (214) Write a value of $\int e^{3 \log x} (x^4) dx$.

(215) Write the position vector of a point dividing the line segment joining the points A and B with position vectors \vec{a} & \vec{b} externally in the ratio 1:4, where $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + \hat{k}$.

(216) Find the angle between the line $\frac{x+2}{1} = \frac{y-5}{-2} = \frac{z+9}{-3}$ and the plane $2x - 3y + 6z + 11 = 0$.

(217) Let \vec{a} and \vec{b} be two vectors such that $|\vec{a}| = 3$, $|\vec{b}| = \frac{\sqrt{2}}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector. Then

what is the angle between $\vec{a} \times \vec{b}$?

(218) Show that the line $\vec{r} = 2\hat{i} + 3\hat{j} + \lambda(7\hat{i} - 5\hat{k})$ lies in the plane $\vec{r} \cdot (5\hat{i} - 3\hat{j} + 7\hat{k}) = 1$.

(219) Find the value of c and d if the plane $2x + 4y - cz + d = 0$ will contain the line $\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-1}{4}$.

(220) If B is a skew symmetric matrix, write whether the matrix (ABA') is symmetric or skew symmetric.

(221) Find the position vector of point which is three fifth of the way from $(3, 4, 5)$ to $(-2, -1, 0)$ by vector method.

(222) If \vec{a} & \vec{b} are two unit vectors such that $|\vec{a} + \vec{b}| = \sqrt{3}$, find $(2\vec{a} - 5\vec{b}) \cdot (3\vec{a} + \vec{b})$.

(223) Find the order and degree of the differential equation, $x^2 \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx}\right)^3 + 5 = 0$.

(224) The Cartesian equation of a line are $6x - 2 = 3y + 1 = 2z - 2$. Find the direction ratio of the line.

(225) Evaluate $\int_1^3 \frac{\sqrt{4-x}}{\sqrt{x} + \sqrt{4-x}} dx$.

(226) Let \vec{a} and \vec{b} be two vectors such that $|\vec{a}| = 3$, $|\vec{b}| = \frac{\sqrt{2}}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector. Then

what is the angle between $\vec{a} \times \vec{b}$?

(227) Find the perpendicular distance from $(2, 5, 6)$ on XY plane.

(228) Write the range of one branch of $\sin^{-1} x$, other than the principal Branch.

(229) On expanding by first row, the value of a third order determinant is $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$. Write the expression for its value on expanding by 2nd column. Where A_{ij} is the cofactor of element a_{ij}

(230) For two non zero vector \vec{a} and \vec{b} write when $|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$ holds. Ansa and b are like vector or one of them is zero.

(231) Find whether the relation R in the set $A = \{1, 2, 3\}$ given by $R = \{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$ is transitive.

- (232) For what value of k, the matrix $A = \begin{bmatrix} 4 & 3-k \\ 1 & 2 \end{bmatrix}$ is not invertible ?
- (233) If A is symmetric matrix, then find whether $B^T AB$ is symmetric. B^T is the transpose of matrix B.
- (234) Find x if $[x \ -3 \ 2] \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} = 0$
- (235) State Rolle's theorem.
- (236) Find the order and degree of the differential equation, $x^2 \frac{d^2 y}{dx^2} + x \left(\frac{dy}{dx} \right)^3 + 5 = 0$.
- (237) If the given lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, find k.
- (238) Evaluate : $\int [1 + 2 \cot x (\cot x + \operatorname{cosec} x)]^{1/2} dx$.
- (239) Determine the binomial distribution whose mean is 9 and whose standard deviation is $3/2$.
- (240) Find the value of x if the area of Δ is 5 square cms with vertices $(x+1, 4)$, $(2, -6)$ and $(5, 4)$.
- (241) If $\vec{AB} = 4\hat{i} + 5\hat{j} - 3\hat{k}$ and the position vector of point B is $3\hat{i} - \hat{j} + 2\hat{k}$, find the position vector of point A.
- (242) If A and B are independent events such that $P(A \cup B) = 0.8$, $P(A) = 0.5$, then find $P(B)$.
- (243) If the matrix A is both symmetric and skew symmetric, then what is the matrix A ?
- (244) At what point on the curve $y^2 = 2x - 1$ does the ordinate decrease as the same rate as the abscissa increase ?
- (245) Find the direction ratios of the line : $2x = \frac{1-y}{2} = \frac{3z+4}{6}$.
- (246) Give an example of a relation which is reflexive and symmetric but not transitive.
- (247) The equation of a line is $\frac{2x-5}{4} = \frac{y+4}{3} = \frac{6-z}{6}$. Find the direction cosines of a line parallel to this line.
- (248) Write the principal value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$.
- (249) Write the range of the principal branch of $\sec^{-1}(x)$ defined on the domain $R - (-1, 1)$.
- (250) Find x if $\begin{vmatrix} 3 & 4 \\ -5 & 2 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ -5 & 3 \end{vmatrix}$.
- (251) If A is a square matrix of order 3 such that $|\operatorname{adj}A| = 64$. find $|A|$.
- (252) If A is a square matrix satisfying $A^2 = 1$, then what is the inverse of A?

- (253) If $f(x) = \sin x^0$, find $\frac{dy}{dx}$.
- (254) What is the degree of the following differential equation? $y \frac{d^2y}{dx} + \left(\frac{dy}{dx}\right)^3 = x \left(\frac{d^3y}{dx^3}\right)^2$.
- (255) If \vec{a} and \vec{b} represent the two adjacent sides of a parallelogram, then write the area of parallelogram in terms of \vec{a} and \vec{b} .
- (256) Find the angle between two vectors \vec{a} and \vec{b} if $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{a} \times \vec{b}| = 6$.
- (257) Find the direction cosines of a line, passing through origin and lying in the first octant, making equal angles with the three coordinate axes.
- (258) If A is square matrix of order 3 such that $|adjA| = 64$ Find $|A|$.

PART – B

- (259) Find $\frac{d^2y}{dx^2}$ when : $x = a(\cos t + t \sin t), y = a(\sin t - t \cos t)$.
- (260) In an examination, 8 questions of true- false type are asked. A student tosses a fair die to determine his answer to each question. If the die show odd prime, he answers true otherwise, he answers false. Find that the probability that he answers at most 6 questions true.
- (261) Given that vectors A, B, C form a triangle such that $A = B + C$. Find a,b,c,d such that the area of the triangle is $5\sqrt{6}$ where $A = ai+bj+ck, B = di+3j+4k, C = 3i +j - 2k$.
- (262) Examine the continuity of the function f defined by $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ at $x = 0$.
- (263) Discuss the differentiability of $f(x) = \begin{cases} 1-x & x < 1 \\ (1-x)(2-x) & 1 \leq x \leq 2 \\ 3-x & x > 2 \end{cases}$ at $x = 1$ & $x = 2$.
- (264) If $y = (\sin^{-1} x)^2 + (\cos^{-1} x)^2$ then prove that $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 4$.
- (265) A line makes angles α, β, γ and δ with the diagonals of a cube, prove that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = \frac{4}{3}$.
- (266) Solve the following differential equation : $(1 + y + x^2 y)dx + (x + x^3)dy = 0$, where $y = 0$ when $x = 1$.
- (267) Evaluate: $\int_0^{\pi/2} x \cot x dx = \frac{\pi}{2} \log 2$.
- (268) If $y = (\log x)^{\cos x} + \frac{x^2 + 1}{x^2 - 1}$, find $\frac{dy}{dx}$

- (269) Using the properties of determinants, show that : $\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = (a^3 - 1)^2$
- (270) For the curve $y = 4x^3 - 2x^5$ find all the points at which tangent passes through the origin .
- (271) Using properties of definite integral, evaluate: $\int_0^\pi \frac{xdx}{4 - \cos^2 x}$.
- (272) Find a vector of magnitude 5 units perpendicular to each of the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$
- (273) A variable plane which remains at a constant distance of 9 units from the origin, cuts the coordinate axes at the point A, B and C. show that the locus of the centroid of ΔABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{9}$.
- (274) If $x = a \left\{ \cos t + \log \tan \frac{t}{2} \right\}$ and $y = a \sin t$, find $\frac{dy}{dx}$.
- (275) Using properties of definite integral, prove the following: $\int_0^\pi \frac{x \tan x}{\sec x \cos ecx} dx = \frac{\pi^2}{4}$.
- (276) Evaluate : $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$.
- (277) If X follows binomial distribution with mean 3 and variance $\frac{3}{2}$, find $P(X \leq 5)$.
- (278) Show that the line $\vec{r} = 4\hat{i} - 7\hat{k} + \lambda(4\hat{i} - 2\hat{j} + 3\hat{k})$ is parallel to the plane $\vec{r} \cdot (5\hat{i} + 4\hat{j} - 4\hat{k}) = 7$. Also find the distance from point to plane .
- (279) Verify Roll'es theorem for the function $f(x) = (x-a)^m (x-b)^n$ on the internal $[a,b]$ where m, n are positive integers.
- (280) A company has estimated that the probabilities of success for three products introduced in the market are $\frac{1}{3}, \frac{2}{5}$ & $\frac{2}{3}$ respectively. Assuming independence, find (i) the probability that the three products are successful (ii) the probability that none of the products is successful.
- (281)
- (282) Differentiate w.r.t x : $x^x + x^a + a^x + a^a$, for some fixed $a > 0$ and $x > 0$.
- (283) Find the approximate value of $\sqrt{0.0037}$, using differentials.
- (284) Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 2x^3 - 7$, for $x \in \mathbb{R}$ is bijective.
- (285) Let $f, g : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = |x|$ and $g(x) = [x]$ where $[x]$ denotes the greatest integer less than or equal to x . Find $f \circ g \left(\frac{5}{2} \right)$ and $g \circ f \left(-\sqrt{2} \right)$.

- (286) Prove that $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$.
- (287) If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$. Show that $A^2 - 5A - 14I = 0$. Hence find A^{-1} .
- (288) Show that $f(x) = |x - 3|, \forall x \in R$, is continuous but not differentiable at $x = 3$.
- (289) If $\tan\left(\frac{x^2 - y^2}{x^2 + y^2}\right) = a$, then prove that $\frac{dy}{dx} = \frac{y}{x}$.
- (290) Verify Rolle's Theorem for the function f , given by $f(x) = e^x(\sin x - \cos x)$ on $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$.
- (291) Using differentials, find the approximate value of $\sqrt{25.2}$.
- (292) Two equal sides of an isosceles triangle with fixed base 'a' are decreasing at the rate of 9 cm/second. How fast is the area of the triangle decreasing when the two sides are equal to 'a'.
- (293) Evaluate $\int_{-1}^1 |x \cos(\pi x)| dx$.
- (294) Find the intervals in which the function $f(x) = \sin\left(2x + \frac{\pi}{4}\right), 0 \leq x \leq 2\pi$ is (i) increasing (ii) decreasing .
- (295) Solve the following differential equation : $ye^{\frac{x}{y}} dx = \left(xe^{\frac{x}{y}} + y\right) dy$.
- (296) Solve the following differential equation : $(1+y+x^2 y) dx + (x+x^3) dy = 0$, where $y=0$ when $x=1$.
- (297) If \vec{a}, \vec{b} and \vec{c} are three unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and angle between \vec{b} and \vec{c} is $\frac{\pi}{6}$, prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$
- (298) Find the values of a and b so that the function $f(x) = \begin{cases} ax^2 + b, & x < 2 \\ 2 & x = 2 \\ 2ax - b, & x > 2 \end{cases}$ may be continuous.
- (299) Show that the four point $(0, -1, -1), (4, 5, 1), (3, 9, 4)$ and $(-4, 4, 4)$ are coplanar. Also, find the equation of the plane containing them.
- (300) A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed three times ,find the probability distribution of number of tails.
- (301) How many time must a man toss a fair coin, so that the probability of having at least one head is more than 80%?
- (302) The random variable X has a probability distribution $P(X)$ of the following form, where K is sqme number :

$$P(X) = \begin{cases} k, & \text{if } X = 0 \\ 2k, & \text{if } X = 1 \\ 3k, & \text{if } X = 2 \\ 0, & \text{if otherwise} \end{cases}$$

Find the value of (i) k. (ii) $P(X < 2)$ (iii) $P(X \leq 2)$ (iv) $P(X \geq 2)$.

(303) Find a unit vector perpendicular to the plane ABC where A, B and C are the points (3, -1, 2), (1, -1, -3) and (4, -3, 1) respectively.

(304) Let $A = \{2, 3, 4, 5, 6, 7, 8, 9\}$. Let R be the relation on A defined by $\{(x, y) : x \in A \text{ \& } x \text{ divides } y\}$. Find (i) R (ii) Domain of R (iii) Range of R (iv) R^{-1} state whether or not R^{-1} , is (a) Reflexive (b) symmetric (c) transitive.

(305) Find the particular solution of the differential equation $(x dy - y dx) y \sin\left(\frac{y}{x}\right) = (y dx + x dy) x \cos\left(\frac{y}{x}\right)$, given that $y = \pi$ when $x = 3$.

(306) let $A = N \times N$ and $*$ be an binary operation defined by $(a, b) * (c, d) = (ac, bd) \forall a, b, c, d \in N$ on the set R, then (i) Prove that $*$ is a binary operation on N (ii) Is $*$ commutative? (iii) Is associative (iv) Find the identity element for $*$ on $N \times N$ if any.

(307) Evaluate: $\int_0^{3/2} |x \cos \pi x| dx$.

(308) Let N be the set of all natural numbers and R be the relation in $N \times N$ defined by (a, b) R (c, d) if $ad = bc$. Show that R is an equivalence relation.

(309) Prove that $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2} \cos^{-1}\left(\frac{3}{5}\right)$.

(310) Solve for x : $\sin^{-1}(1-x) - 2 \sin^{-1} x = \frac{\pi}{2}$.

(311) Using properties of determinants, prove that :
$$\begin{vmatrix} a^2 + 1 & ab & ac \\ ab & b^2 + 1 & bc \\ ca & cb & c^2 + 1 \end{vmatrix} = (1 + a^2 + b^2 + c^2)$$

(312) An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that (i) all will bear 'X' mark. (ii) not more than 2 will bear 'Y' mark (iii) at least one ball will bear 'Y' mark (iv) the number of balls with 'X' mark and 'Y' mark will be equal.

(313) The sum and the product of the mean and variance of a binomial distribution are 24 and 128 respectively. Find the distribution.

(314) If $y = e^{ax} \sin bx$. Show that $\frac{d^2 y}{dx^2} - 2a \frac{dy}{dx} + y(a^2 + b^2) = 0$.

(315) For what value of a and b, the function f defined as:

$$f(x) = \begin{cases} 3ax+b, & \text{if } x < 1 \\ 11, & \text{if } x = 1 \\ 5ax-2b, & \text{if } x > 1 \end{cases} \text{ is continuous at } x=1.$$

(316) Find the equation of the line passing through the point P(4, 6, 2) and the point of intersection of the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z+1}{7}$ and the plane $x + y - z = 8$.

(317) Using elementary transformation, find the inverse of the following matrix $\begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$

(318) Solve for x: $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1} \frac{8}{31}$.

(319) If $y = x^x$ then prove that $\frac{d^2 y}{dx^2} - \frac{1}{y} \left(\frac{dy}{dx} \right)^2 - \frac{y}{x} = 0$.

(320) Let $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x < \frac{\pi}{2} \\ a & \text{if } x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi-2x)^2} & \text{if } x > \frac{\pi}{2} \end{cases}$. If $f(x)$ be a continuous function at $x = \frac{\pi}{2}$, find a and b.

(321) If $y = \left[\log \left(x + \sqrt{1+x^2} \right) \right]^2$, show that $(1+x^2) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - 2 = 0$.

(322) Find the distance between the points with position vectors $-\hat{i}-5\hat{j}-10\hat{k}$ and the points of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ with the plane $x - y + z = 5$.

(323) Two balls are drawn one by one with replacement from a bag containing 4 red and 6 black balls. Find the probability distribution of 'number of red balls'. Also find mean of distribution.

(324) Consider the function, $f : R^+ \rightarrow [4, \infty]$ defined by $f(x) = x^2 + 4$. Prove that f is invertible. Also find f^{-1} .

(325) If $x^y + y^x = a^b$ find $\frac{dy}{dx}$.

(326) If $x = a(\cos t + t \sin t)$ and $y = b(\sin t - t \cos t)$, find $\frac{d^2 y}{dx^2}$.

(327) Find the matrix P satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

- (328) If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} \times \vec{b} = \vec{c}$, $\vec{b} \times \vec{c} = \vec{a}$ prove that $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular and $|\vec{a}| = |\vec{c}|$ and $|\vec{b}| = 1$.
- (329) Find the intervals in which the function $f(x) = \sin 3x$; $0 \leq x \leq \frac{\pi}{2}$ (i) is increasing (ii) is decreasing.
- (330) Prove that: $4 \tan^{-1} \frac{1}{5} - \tan^{-1} \frac{1}{70} + \tan^{-1} \frac{1}{99} = \frac{\pi}{4}$.
- (331) Prove that: $\tan^{-1} \left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right) = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x$, $-\frac{1}{\sqrt{2}} \leq x \leq 1$.
- (332) If $y = \frac{\log x}{x}$, Show that $\frac{d^2y}{dx^2} = \frac{2 \log x - 3}{x^3}$.
- (333) If $y = a^{t + \frac{1}{t}}$ and $x = \left(t + \frac{1}{t} \right)^a$ for positive constant a , find $\frac{dy}{dx}$.
- (334) Find the equation of the line passing through the point P(4,6,2) and the point of intersection of the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z+1}{7}$ and the plane $x + y - z = 8$.
- (335) Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Consider the function $f: A \rightarrow B$ defined by $f(x) = \left(\frac{x-2}{x-3} \right)$. Show that f is bijective.
- (336) Find the equation of tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line $4x - 2y + 5 = 0$.
- (337) Find the intervals in which the following function is strictly increasing or strictly decreasing $f(x) = 20 - 9x + 6x^2 - x^3$.
- (338) For the curve $y = 4x^3 - 2x^5$, find all point at which the tangent passes through origin.
- (339) Evaluate: $\int \frac{\sin x + \cos x}{\sqrt{\sin x \cdot \cos x}} dx$.
- (340) Evaluate: $\int e^x \left(\frac{x^2 + 1}{(x+1)^2} \right) dx$.
- (341) Form the differential equation of the family of circles having radii 3.
- (342) Solve the following differential equation: $\sqrt{1+x^2+y^2+x^2y^2} + xy \frac{dy}{dx} = 0$.
- (343) If the sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.
- (344) Find whether the lines $\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + \lambda(2\hat{i} + \hat{j})$ and $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} + \hat{j} - \hat{k})$ intersect or not. If intersecting, find their point of intersection.

- (345) Three balls are drawn one by one without replacement from a bag containing 5 white and 4 green balls .find the probability distribution of number of green balls drawn.
- (346) Define a binary operation * one the set $\{0,1,2,3,4,5\}$ as $a * b = \begin{cases} a+b, a+b < 6 \\ a+b-6, a+b \geq 6 \end{cases}$. Show that zero is the identity for this operation and each element a of the set is invertible with $6 - a$ being the inverse of a.
- (347) Evaluate: $\tan \left[\frac{1}{2} \sin^{-1} \left(\frac{2a}{1+a^2} \right) + \frac{1}{2} \cos^{-1} \left(\frac{1-a^2}{1+a^2} \right) \right]$.
- (348) Prove that $\tan \left[\frac{\pi}{4} + \frac{1}{2} \cos^{-1} \frac{a}{b} \right] + \tan \left[\frac{\pi}{4} - \frac{1}{2} \cos^{-1} \frac{a}{b} \right] = \frac{2b}{a}$
- (349) Find the values of a and b if $f(x) = \begin{cases} 2ax + b, x < 2 \\ 19, x = 2 \\ 3a - 2bx, x > 2 \end{cases}$ is continuous at $x = 2$.
- (350) Find the vector equation of the line parallel to the line $\frac{x-1}{5} = \frac{3-y}{2} = \frac{z+1}{4}$ and passing through the point $(3, 0, -4)$. Also find the distance between two lines .
- (351) Find the shortest distance between the lines whose vector equations are $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ & $\vec{r} = (2\hat{i} + 4\hat{j} + 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 8\hat{k})$. Also find the angle between two lines .
- (352) For any two vectors \vec{a} & \vec{b} , show that $(1 + |\vec{a}|^2)(1 + |\vec{b}|^2) = (1 - \vec{a} \cdot \vec{b})^2 + |\vec{a} + \vec{b} + (\vec{a} \times \vec{b})|^2$.
- (353) If $\sin(\sin^{-1} \frac{1}{5} + \cos^{-1} x) = 1$, then find the value of x.
- (354) Prove that the lines $\frac{X+4}{3} = \frac{Y+6}{5} = \frac{Z-1}{-2}$ and $3x-2y+z+5=0; 2x+3y+4z-4=0$ are coplanar . Also write the equation of plane in which they lie.
- (355) The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm/ sec . How fast is the area decreasing when the two equal sides are equal to the base?
- (356) Prove that $\left| \vec{a} \times \vec{b} \right|^2 = \left| \vec{a} \right|^2 \left| \vec{b} \right|^2 - (\vec{a} \cdot \vec{b})^2$ and hence find $\vec{a} \cdot \vec{b}$, if $\left| \vec{a} \times \vec{b} \right| = 16, \left| \vec{a} \right| = 10$ and $\left| \vec{b} \right| = 2$.
- (357) If $y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$ Prove that $\sin y = \tan^2 \frac{x}{2}$.
- (358) Prove that : $[\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}] = [\vec{a} \quad \vec{b} \quad \vec{c}]^2$.
- (359) For any vector \vec{a} prove that $\left| \vec{a} \times \hat{i} \right|^2 + \left| \vec{a} \times \hat{j} \right|^2 + \left| \vec{a} \times \hat{k} \right|^2 = 2 \left| \vec{a} \right|^2$.
- (360) Evaluate: $\int \frac{e^{\tan^{-1} x}}{(1+x^2)^2} dx$.

- (361) Prove that $y^2 = a(b^2 - x^2)$ is a solution of differential equation $xy \frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2 - y \frac{dy}{dx} = 0$.
- (362) By examining the chest X-ray, the probability that T.B. is detected when a person is actually suffering is 0.99. The probability that the doctor diagnoses is incorrectly that a person has T.B. on the basis of x-ray is 0.001. In a certain city, 1 in 1000 persons suffers from TB. A person is selected at random and is diagnosed to have T.B. What is the chance that he actually has T.B.?
- (363) Find the vector equation of the following plane in scalar product form $\vec{r} = (i - j) + \lambda(i + j + k) + \mu(i - 2j + 3k)$.
- (364) Show that the relation R in the set $A = \{x; x \in \mathbb{Z}, 0 \leq x \leq 12\}$ given by $R = \{(a, b) : |a - b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1.
- (365) Solve for x : $2 \tan^{-1}(\sin x) = \tan^{-1}(2 \sec x), 0 < x < \frac{\pi}{2}$.
- (366) Show that : $\tan \frac{1}{2} \left[\sin^{-1} \frac{2x}{1+x^2} + \cos^{-1} \frac{1-y^2}{1+y^2} \right] = \frac{x+y}{1-xy}, |x| < 1, y > 0, xy < 1$.
- (367) If none of a, b and c is zero, using properties of determinants.
- (368) Prove that : $\begin{vmatrix} -bc & b^2 + bc & c^2 + bc \\ a^2 + ac & -ac & c^2 + ac \\ a^2 + ab & b^2 + ab & -ab \end{vmatrix} = (bc + ca + ab)^3$.
- (369) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.
- (370) If $y = (x + \sqrt{x^2 + 1})^m$, then show that $(x^2 + 1) \frac{d^2y}{dx^2} + x \frac{dy}{dx} - m^2 y = 0$.
- (371) Find all the points of discontinuity of the function $f(x) = (x^2)$ on $[1, 2]$ where $[]$ denotes the greatest integer function.
- (372) Differentiate $\sin^{-1}(2x\sqrt{1-x^2})$ w.r.t. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$.
- (373) Evaluate : $\int \frac{1}{\cos(x-a)\cos(x-b)} dx$.
- (374) Evaluate : $\int x(\log x)^2 dx$.
- (375) Evaluate : $\int \frac{x}{x^3 - 1} dx$.
- (376) Using properties of definite integrals, evaluate: $\int_0^\pi \frac{x dx}{4 - \cos^2 x}$.

- (377) The dot products of a vector with the vectors $\hat{i} - 3\hat{k}$, $\hat{i} - 2\hat{k}$ and $\hat{i} + \hat{j} + 4\hat{k}$ are 0, 5, and 8 respectively. Find the vector.
- (378) Find the equation of plane passing through the point (1, 2, 1) and perpendicular to the line joining the points (1, 4, 2) and (2, 3, 5). Also, find the perpendicular distance of the plane from the origin.
- (379) Find the equation of the perpendicular drawn from the point P(2, 4, -1) to the line. $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$.
- (380) A company sells two different products A and B. The two products are produced in a common production process which has a total capacity of 500 man hours. It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B. The demand in the market shows that the maximum number of units of A that can be sold is 70 and that of B is 125. Profit on each unit of A is Rs. 20 and on B is Rs. 15. How many units of A and B should be produced to maximize the profit. Form an L.P.P. and solve it graphically.
- (381) If a unit vector \vec{a} makes angles $\frac{\pi}{4}$ and $\frac{\pi}{3}$ with x-axis and y-axis respectively and an acute angle θ with z-axis, then find θ and the (scalar and vector) components of \vec{a} along the axes.
- (382) If \vec{a}, \vec{b} & \vec{c} are three vectors such that $\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a}; \vec{c} \times \vec{a} = \vec{b}$, prove that $\vec{a}, \vec{b}, \vec{c}$ are right handed system of orthogonal unit vector.
- (383) A line passes through the point (-1, 3, 7) and is perpendicular to the lines $\vec{r} = (2\hat{i} - 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + \hat{k})$ and $\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \mu(7\hat{j} - 5\hat{k})$. Obtain its equation.
- (384) If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are any four vectors, show that $(\vec{a} \times \vec{b}) (\vec{c} \times \vec{d}) = \begin{vmatrix} \vec{a} & \vec{c} & \vec{a} & \vec{d} \\ \vec{b} & \vec{c} & \vec{b} & \vec{d} \end{vmatrix}$.
- (385) If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, show that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$ where $\vec{a} \neq \vec{d}$ & $\vec{b} \neq \vec{c}$.
- (386) Solve for x : $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$.
- (387) If $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$, prove that $\frac{dy}{dx} = \frac{x-1}{2x(x+1)}$.
- (388) Find the inverse of the matrix $\begin{bmatrix} -1 & 2 & 5 \\ 2 & -3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$ using elementary transformation.
- (389) Suppose the reliability of HIV test is specified as follows. Of people having HIV, 90% of the test detects the disease but 10% go undetected. Of people not having HIV, 99% of the test is judged HIV -ve but 1% are diagnosed as showing HIV + ve. From a large population of which only 0.1% has HIV, one person is selected at random, given

the HIV test, and the pathologist reports as HIV +ve. What is the probability that the person actually has HIV?

(390) Find the area of the region in the first quadrant enclosed by the line $y = x$ and the circle $x^2 + y^2 = 32$ above x axis ..

(391) If $y = x^x$ then prove that $\frac{d^2 y}{dx^2} - \frac{1}{y} \left(\frac{dy}{dx} \right)^2 - \frac{y}{x} = 0$.

(392) Suppose a girl throws a die . If she gets a 5 or 6 , she tosses a coin three times and note the number of heads . If she gets a 1 , 2, 3 or 4 , she tosses a coin once and notes whether a heads or tail is obtained . If she obtained exactly one head ;what is the probability that she threw 1 , 2 , 3 or 4 with the die .

(393) If $f(x) = \begin{cases} \frac{x-4}{|x-4|} + a & x < 4 \\ a+b & x = 4 \\ \frac{x-4}{|x-4|} + b & x > 4 \end{cases}$ Determine the values of a and b so that $f(x)$ is continuous at

$x=4$.

(394) Let $\vec{A} = 2\hat{i} + \hat{k}$, $\vec{B} = \hat{i} + \hat{j} + \hat{k}$ & $\vec{C} = 4\hat{i} - 3\hat{j} + 7\hat{k}$. Determine a vector \vec{R} satisfying $\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$ and $\vec{R} \cdot \vec{A} = 0$.

(395) Let $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$ and $f, g: A \rightarrow B$ be functions defined by $f(x) = x^2 - x$, $x \in A$ and $g(x) = 2 \left| x - \frac{1}{2} \right| - 1$, $x \in A$ are f and g equal. Justify your answer.

(396) Prove that $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2 + b^2 + c^2)$.

(397) Show that the function $f(x) = \begin{cases} \frac{e^x - 1}{x} & \text{if } x \neq 0 \\ e^x + 1 & \text{if } x = 0 \end{cases}$ is discontinuous at $x = 0$.

(398) If $\vec{a} = \vec{i} + \vec{j} + \vec{k}$, $\vec{c} = \vec{j} - \vec{k}$ are given vectors, find a vector \vec{b} satisfying the equation $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 3$.

(399) Let $\vec{a} = 2\vec{i} + \vec{k}$, $\vec{b} = \vec{i} + \vec{j} + \vec{k}$ and $\vec{c} = 4\vec{i} - 3\vec{j} + 3\vec{k}$ be three vectors , find a vector \vec{r} which satisfies $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$.

(400) Solve the differential equation $\frac{dy}{dx} - 3y \cot x = \sin 2x$; $y = 2$ when $x = \frac{\pi}{2}$.

(401) Find the equation of tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line $4x - 2y + 5 = 0$.

(402) If a, b, c is real numbers and $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$. Show that either $a + b + c = 0$ or $a = b = c$.

(403) Find the shortest distance between the lines whose vector equations are $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ & $\vec{r} = (2\hat{i} + 4\hat{j} + 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 8\hat{k})$. Also find the angle between two lines.

(404) A plane meets the coordinate axis in A, B, C such that the centroid of triangle ABC is the point (p, q, r). Prove that the equation of plane is $\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 3$.

(405) If $y = (\log x)^{\cos x} + \frac{x^2 + 1}{x^2 - 1}$, find $\frac{dy}{dx}$.

(406) Prove that the angle between any two diagonal of cuboids is $\cos^{-1}\left(\frac{a^2 \pm b^2 \pm c^2}{a^2 + b^2 + c^2}\right)$.

(407) Find the point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance $3\sqrt{2}$ from the point (1, 2, 3).

(408) Dot product of a vector with vectors $3\hat{i} - 5\hat{k}$, $2\hat{i} + 7\hat{j}$, and $\hat{i} + \hat{j} + \hat{k}$ are respectively -1, 6 and 5. Find the vector.

(409) For any vector \vec{a} , prove that $\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$.

(410) Show that : $2 \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7} = \sec^{-1} \frac{\sqrt{34}}{5} + \cos^{-1} \sqrt{17}$.

(411) Obtain the differential equation of the family of Hyperbola of the family of Hyperbola having foci on y-axis and centre at the origin.

(412) Solve the following differential equation $(1-x^2)\frac{dy}{dx} - xy = x^2$, given $y=2$ when $x=0$.

(413) A, B and C play a game and chances of their winning it in an attempt are $\frac{2}{3}$, $\frac{1}{2}$ and $\frac{1}{4}$ respectively. A has the first chance, followed by B and then C. The cycle is repeated till one of them wins the game. Find their respective chances of winning the game.

(414) Evaluate $\int_{-5}^0 f(x)dx$, where $f(x) = |x| + |x+2| + |x+5|$.

(415) If a line makes an angle α, β, γ and δ with the four diagonals of a cube prove that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = \frac{4}{3}$.

(416) A right circular cone is circumscribed about a right circular cylinder of radius 'r' and altitude 'h'. show that the volume of the cone is least, when

- (i) the altitude of the cone is equal to 3 times the altitude of the cylinder.
- (ii) The radius of the cone is equal to 3/2 times the radius of the cylinder.

- (417) The sum of the surface areas of a rectangular parallelopiped with sides x , $2x$ and $\frac{x}{3}$ and a sphere given to be constant. Prove that the sum of their volume is minimum, if x is equal to three times the radius of the sphere. Find the minimum value of the sum of the volumes.
- (418) Make a rough sketch of the region, given below and find its area using integration. $\{(x, y) : 0 \leq y \leq x^2 + 3; 0 \leq y \leq 2x + 3; 0 \leq x \leq 3\}$ (Ans. $y\sqrt{1-x^2} = -\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x + 2$)
- (419) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ & $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Find their point of intersection. Also find the equation of plane contain them. (Ans. $\frac{50}{3}$)
- (420) Find the foot and length of perpendicular from the point $(2, 3, 4)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also find image of the point $(2, 3, 4)$.
- (421) Find the shortest distance between the lines $x+4 = y-4 = \frac{z-1}{-1}$ & $\frac{x+3}{2} = \frac{y+8}{3} = \frac{z+3}{3}$. Also find the equations of this line.
- (422) Let $*$ be a binary operation on $Q \times Q$. If $(a,b) * (c,d) = (ac, b + ad)$; $(a,b), (c,d) \in Q \times Q$. Prove that (i) $*$ is closed to binary operation on $Q \times Q$ (ii) $*$ is commutative on $Q \times Q$ (iii) $*$ is associative on $Q \times Q$ (iii) Find the identity element with respect to operation $*$ on $Q \times Q$ if any. Also find the invertible element on $Q \times Q$.
- (423) Find the equations of the line which intersects the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ & $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and passes through the point $(1, 1, 1)$.
- (424) Prove(by vector method) that the parallelograms on the same base and between the same parallel are equal in area.
- (425) Find the value of λ so that the four points with position vectors $-\hat{j} + \hat{k}, 2\hat{i} - \hat{j} - \hat{k}, \hat{i} + \lambda\hat{j} + \hat{k}$ and $3\hat{j} + 3\hat{k}$ are coplanar. Also write the equation of plane for this value of λ .
- (426) Evaluate : $\int \frac{\tan \theta + \tan^3 \theta}{1 + \tan^3 \theta} dx$..
- (427) Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by $x = 0, x = 4, y = 4, y = 0$ into three equal parts.
- (428) Evaluate: $\int_0^{\pi/2} \frac{\cos x}{1 + \cos x + \sin x} dx$.
- (429) Find the area of the smaller region bounded by the ellipse $9x^2 + 25y^2 = 225$ and the line $3x + 5y = 15$.

(430) Find the area bounded by the curve $y = (x-1)(x - 2) (x-3)$ lying between the ordinates $x = 0$ and $x = 3$.

(431) Solve the differential equation: $\frac{d^2x}{dy^2} = y \sin^2 y$.

(432) Let $N \times N$ be the set of ordered pairs of natural numbers . Also let R be the relation in $N \times N$, defined by $(a,b)R(c,d) \Leftrightarrow ad = bc$.Show that R is an equivalence relation .

(433) Find a unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ & $\vec{a} - \vec{b}$ where $\vec{a} = 3i + 2j + 2k$ and $\vec{b} = i + 2j - 2k$.

(434) Using integration find the area of the region bounded by the parabola $x^2 + y^2 = 16$,x axis and the line $y = x$ in the first quadrant . .(Ans. 2π sq unit

(435) Let * be the binary operation such that $Q \times Q = Q$ defined by $a * b = a + b - ab$; $a, b \in Q - \{1\}$. Show that * is (i) associative, (ii) commutative.(Ans. (i) * is associative. (ii) * is commutative.)

(436) Show that $f : N \rightarrow N$ defined by $f(x) = \left\{ \begin{array}{l} \frac{n+1}{2}, n \text{ is odd} \\ \frac{n}{2}, n \text{ is even} \end{array} \right\}$ is many one on to function .

(437) Show that $f : \{-1,1\} \rightarrow R$; given by $f(x) = \frac{x}{x+2}, x \neq -2$ is one – one. Find the inverse of the function $f : \{-1,1\} \rightarrow R$.(Ans. $f^{-1}(x) = \frac{2x}{1-x}$)

(438) Let a pair of dice be thrown and the random variable X be the sum of the numbers that appear on the two dice . Find the mean or expectation of X .

(439) The sum and the product of the mean and variance of a binomial distribution are 24 and 128 respectively. Find the distribution.

(440) Using integration, find the area of the region in the first quadrant enclosed by x-axis, the line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

(441) For any vector \vec{a} prove that $\left| \vec{a} \times \hat{i} \right|^2 + \left| \vec{a} \times \hat{j} \right|^2 + \left| \vec{a} \times \hat{k} \right|^2 = 2 \left| \vec{a} \right|^2$.

(442) A firm manufactures two types of product A and B and sells them at a profit of Rs 5 per unit of type A and Rs. 3 per unit of type B. Each product is processed on two machines M_1 & M_2 .One out of type A requires one minute of processing time on M_1 and two minutes of processing time on M_2 , whereas one minute of type B requires one minute of processing time on M_1 and one minute on M_2 . Machine M_1 & M_2 are respectively available for at most 5 hour and 6 hours in a day . Find out how many units of each type of product should the firm produce a day in order to maximize the profit. Solve the graphically.

(443) Using integration, find the area of the two parabolas $4y^2 = 9x$ & $3x^2 = 16y$. Also find the angle between two curves.

(444) In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $5/6$. What is the probability that he will knock down fewer than 2 hurdles?

(445) Prove that
$$\begin{vmatrix} 3a & -a+b & -a+c \\ a-b & 3b & c-b \\ a-c & b-c & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca).$$

(446) Let X be the random variable which assumes values 0,1,2,3 such that $3P(X=0) = 2P(X=1) = P(X=2) = 4P(X=3)$. Find the probability distribution of X.

(447) Solve for x : $\sin[2 \cos^{-1}\{\cot(2 \tan^{-1} x)\}] = 0$.

(448) If $\cos^{-1} \frac{x}{2} + \cos^{-1} \frac{y}{3} = \theta$, Then prove that $9x^2 - 12xy \cos \theta + 4y^2 = 36 \sin^2 \theta$.

(449) Using properties of determinants, prove that the following:

$$\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 9b^2(a+b)$$

(450) There are three urns A, B and C. Urn A contains 4 white balls and 5 blue balls. Urn B contains 4 white balls and 3 blue balls. Urn C contains 2 white balls and 4 blue balls. One ball is drawn from each of these urns. What is the probability that out of these three balls drawn, two are white balls and one is a blue ball?

(451) If $y = \sin(m \sin^{-1} x)$, prove that $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + m^2y = 0$.

(452) From the differential equation of the family of curves $y = Ae^{3x} + Be^{5x}$, where A and B are arbitrary constant.

(453) A problem of mathematics is given to three students, whose chances of solving it are $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$. What is the probability that (i) the problem is solved (ii) only two of them solved the problem.

(454) Solve the differential equation $x \frac{dy}{dx} = y(\log y - \log x + 1)$.

(455) Let X denote the number of colleges where you will apply after your results and P (X = r) denotes your probability of getting admission in r number of colleges. It is given

that
$$P(X = r) = \begin{cases} kr & r = 0,1 \\ 2kr & r = 2 \\ k(5-r) & r = 3,4 \end{cases}$$
 (a) Find the value of k (b) What is the probability that

you will get admission in exactly two colleges? (c) Find the mean and variance of the probability distribution.

- (456) Using elementary row transformation, find the inverse of $\begin{bmatrix} 5 & -1 \\ 3 & -2 \end{bmatrix}$.
- (457) An aeroplane can carry a maximum of 200 passengers. A profit of Rs 400 is made on each first class ticket and a profit of Rs 300 is made on each second class ticket. The airline reserves at least 20 seats for first class. However, at least four times as many passenger prefer to travel by second class than by first class. Determine how many tickets of each type must be sold to maximize profit for the airline. Form an LPP and solve it graphically.
- (458) Find the condition that the lines $x = a y + b$, $z = c y + d$ and $x = a' y + b$, $z = c' y + d'$ may be perpendicular to each other .
- (459) Let $A = \{ 1, 2, 3, 4 \}$ $B = \{ 3, 5, 7, 9 \}$ $C = \{ 7, 23, 47, 79 \}$ and $f: A \rightarrow B$, $g: B \rightarrow C$ be defined as $f(x) = 2x + 1$ and $g(x) = x^2 - 2$. Express $(g \circ f)^{-1}$ & $f^{-1} \circ g^{-1}$ as the set of ordered pair and verify that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.
- (460) A bag contain 4 white and 6 red balls . Four balls are drawn at random from the bag . Find the probability distribution of the number of white balls . Also find mean and variance .
- (461) If $\cos^{-1} \frac{x}{a} + \cos^{-1} \frac{y}{b} = \alpha$ prove that $\frac{x^2}{a^2} - \frac{2xy}{ab}(\cos \alpha) + \frac{y^2}{b^2} = \sin^2 \alpha$.
- (462) Let $A = \{ 2, 3, 4, 5, 6, 7, 8, 9 \}$. Let R be the relation on A defined by $\{(x, y) : x, y \in A \text{ \& } x \text{ divides } y\}$. Find (i) R (ii) Domain of R (iii) Range of R (iv) R^{-1} state whether or not R^{-1} , is (a) Reflexive (b) symmetric (c) transitive
- (463) If $*$ be the binary operation on the set Q of rational numbers defined by $a * b = \frac{ab}{4}$. For binary operation examine the commutative and associative property. Also find identity element and inverse .
- (464) If f and g be two functions defined by $f(x) = \frac{3x+4}{5x-7} \left(x \neq \frac{7}{5} \right)$ and $g(x) = \frac{7x+4}{5x-3} \left(x \neq \frac{3}{5} \right)$ respectively. Show that $f \circ g = g \circ f$.
- (465) If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices A, B, C of a ΔABC respectively. Find an expansion for the area of ΔABC and hence deduce the condition for the points A, B, C to be collinear .
- (466) Examine the continuity of the function f defined by $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ at $x = 0$.
- (467) Solve the differential equation : $\frac{d^2x}{dy^2} = 1 + \sin y$ given that $dx / dy = 0$ and $y = 0$ when $x = 0$.

(468) If $x = a(\theta + \sin \theta)$ and $y = a(1 - \cos \theta)$, find $\frac{d^2y}{dx^2}$ at $\frac{\pi}{2}$.

(469) A football match may be either won, drawn or lost by the host country's team. So there are three ways of forecasting the result of any match, one correct and two incorrect. Find the probability forecasting at least three correct result for four matches.

(470) Show that the height of a cylinder, which is open at the top having a given surface and greatest volume, is equal to the radius of its base.

(471) Obtain the Binomial distribution if the sum and product of mean and variance Binomial distribution is 24 and 128.

(472) A card from a pack of cards is lost. From the remaining cards of the pack, two cards drawn and are found to be both spades. Find the probability of the lost card being a spade.

(473) Find all the points of discontinuity of the function f defined by $f(x) = \begin{cases} x + 2, & x \leq 1 \\ x - 2, & 1 < x < 2 \\ 0, & x \geq 2 \end{cases}$.

(474) Prove that area of quadrilateral ABCD $ABCD = \frac{1}{2} \vec{AC} \times \vec{BD}$ where AC & BD are the diagonals of quadrilateral ABCD. Also find the area of parallelogram whose two diagonals are $3i + j - 2k$ & $i - 3j + 4k$.

(475) Find the volume of the largest cylinder that can be inscribed in a sphere of radius r cm.

(476) Evaluate $\int_0^2 (x^2 + 3x) dx$ as limit of sum.

(477) Solve the differential equation: $\frac{d^2x}{dy^2} = y \sin^2 y$.

(478) Show that the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect. Find their point of intersection.

(479) It is given that for the function f given by $f(x) = x^3 + bx^2 + ax, x \in [1, 3]$ Rolle's theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$. Find the values of a and b .

(480) If $y = \frac{ax - b}{(x - 1)(x - 4)}$ has a turning point $P(2, -1)$ find the values of a and b and show that y is maximum at P .

(481) Solve the differential equation: $(3xy + y^2)dx + (x^2 + xy)dy = 0$.

(482) Using integration, find the area bounded by the parabola $y^2 = 4x$ and the line $4x - y = 8$.

(483) Prove that $y^2 = a(b^2 - x^2)$ is a solution of differential equation $x \left[y \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^2 \right] - y \frac{dy}{dx} = 0$.

(484) If $x^y + y^x = \log a$, find $\frac{dy}{dx}$.

(485) If $f(x) = \begin{cases} k \cos x & \text{if } x \neq \frac{\pi}{2} \\ \pi - 2x & \\ 3 & \text{if } x = \frac{\pi}{2} \end{cases}$ find the value of k if f is continuous at $x = \frac{\pi}{2}$.

(486) If the function $f(x)$ defined by $f(x) = \begin{cases} \frac{\log(1+ax) - \log(1-bx)}{x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous

at $x = 0$, find k .

(487) Show that $\lim_{x \rightarrow 2} \frac{|x-2|}{x-2}$ does not exist.

(488) In a factory, machines A, B and C produces 25 % and 35% and 40% respectively. Of the total of their output 5%, 4% and 2% are defective. A bolt is drawn at random from the product (i)What is the probability that the bolt drawn is defective? (ii) if the bolt is found to be defective find the probability that this item is produced by the machine B

(489) Two dice are thrown simultaneously . Let X denote the number of sixes find the probability distribution of X . Also find the mean and variance of X , using the probability distribution table .

(490) Show that the differential equation $2ye^{x/y} dx + (y - 2xe^{x/y})dy = 0$ is homogeneous and find its particular solution given that $x = 0$ when $y = 1$.

(491) Find the particular solution of the differential equation $\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x, x \neq 0$ given that $y = 0$, when $x = \frac{\pi}{2}$

(492) Form the differential equation representing the family of ellipse having foci on x-axis and centre at origin.

(493) Find all the local maximum values and local minimum values of the function $f(x) = \sin 2x - x, -\frac{\pi}{2} < x < \frac{\pi}{2}$.

(494) Show that the line $\vec{r} = 4\hat{i} - 7\hat{k} + \lambda(4\hat{i} - 2\hat{j} + 3\hat{k})$ is parallel to the plane $\vec{r} \cdot (5\hat{i} + 4\hat{j} - 4\hat{k}) = 7$. Also find the distance from point to plane .

(495) Find all the points of discontinuity of the function f defined by $f(x) = \begin{cases} x + 2, & x \leq 1 \\ x - 2, & 1 < x < 2 \\ 0, & x \geq 2 \end{cases}$

(496) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$.

(497) Evaluate $\int \frac{\sin 4x - 2}{1 - \cos 4x} e^{2x} dx$.

(498) Find a vector whose magnitude is 3 units and which is perpendicular to the vectors \vec{a} and \vec{b} where $\vec{a} = 3\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 6\hat{i} + 5\hat{j} - 2\hat{k}$.

(499) A, B, C shoot to hit a target. If A hits the target 4 times in 7 trails, B hits it 3 times in 5 trials and C hits it 2 times in 3 trials, what is the probability that the target is hit by at least 2 persons ?

(500) Evaluate : $\int \frac{2 + \sin x}{1 + \cos x} e^{x/2} dx$.

(501) If $x = a \sin pt$ and $y = b \cos pt$, find the value of $\frac{d^2y}{dx^2}$ at $t = 0$.

(502) Prove that the function $f : R - \{3\} \rightarrow R - \{1\}$ given by $f(x) = \frac{x-2}{x-3}$ is bijection.

(503) Show that $\int_0^a \sin^{-1} \sqrt{\frac{x}{a+x}} dx = \frac{a}{2}(\pi - 2)$

(504) Find the equation of tangent and normal to the curve: $x = a \cos t + at \sin t$; $y = a \sin t - at \cos t$, at any point 't' . Also prove that the normal to the curve is at a constant distance from the origin.

(505) Find the equation of the plane containing the lines, $\vec{r} = i + j + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = \hat{i} + \hat{j} + \mu(-\hat{i} + \hat{j} - 2\hat{k})$. Find the distance of this plane from origin and also from the point (1,1,1)

(506) Show that the functions $f(x) = |x+2|$ is a continuous at every $x \in R$ but fails to be differentiable at $x = -2$.

(507) Solve the differential equation : $\frac{d^2x}{dy^2} = 1 + \sin y$ given that $dx / dy = 0$ and $y = 0$ when $x = 0$.

(508) Obtain the differential equation by eliminating a and b from the equation $y = e^x (a \cos x + b \sin x)$.

(509) Find $\frac{dy}{dx}$, if $y = \tan^{-1} \left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right], 0 < |x| < 1$.

(510) The function $f(x)$ is defined as follows: $f(x) = \begin{cases} x^2 + ax + b, & 0 \leq x < 2 \\ 3x + 2, & 2 \leq x \leq 4 \\ 2ax + 5b, & 4 < x \leq 8 \end{cases}$. If is continuous

on $[0,8]$, find the values of a and b .

(511) A letter is known to have come either from TATANAGAR or CALCUTTA. On the envelope, only the two consecutive letters TA are visible. What is the probability that the letter has come from (i) CALCUTTA (ii) TATANAGAR?

(512) Evaluate: $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$.

(513) Evaluate $\int \frac{1}{x^4 + 3x^2 + 16} dx$.

(514) Using limits of sum find the integral of $\int_1^3 (2x^2 + 5) dx$.

(515) A bag contains 50 tickets numbered 1, 2, 3, 50 of which five are drawn at random and arranged in ascending order of magnitude $(x_1 < x_2 < x_3 < x_4 < x_5)$. Find the probability that $x_3 = 30$.

(516) Prove that : $\tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$.

(517) Prove that : $\tan^{-1} \left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right] = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x^2$.

(518) Evaluate: $\int \left\{ \log(\log x) + \frac{1}{(\log x)^2} \right\} dx$.

(519) Solve for x : $\tan^{-1} \left(\frac{x-1}{x-2} \right) + \tan^{-1} \left(\frac{x+1}{x+2} \right) = \frac{\pi}{4}, |x| < 1$.

(520) If $y = \frac{x \sin^{-1} x}{\sqrt{1-x^2}} + \log \sqrt{1-x^2}$, then prove that $\frac{dy}{dx} = \frac{\sin^{-1} x}{(1-x^2)^{3/2}}$.

(521) If $\sqrt{1-x^6} + \sqrt{1-y^6} = a(x^3 - y^3)$, prove that $\frac{dy}{dx} = \frac{x^2}{y^2} \sqrt{\frac{1-y^6}{1-x^6}}$.

(522) If $\sin y = x \sin(a+y)$, prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.

(523) A line passes through the point $(-1,3,7)$ and is perpendicular to the lines $\vec{r} = (2\hat{i} - 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + \hat{k})$ and $\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \mu(7\hat{j} - 5\hat{k})$. Obtain its equation .

(524) Find the equation of the plane through the intersection of planes $x + 3y - 6z = 0$ and $3x - y - 4z = 0$, whose perpendicular distance from the origin equal to 1 .

(525) Solve the differential equation $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0; y(1) = 2$.

(526) If A,B and C are three points with position vectors \vec{a}, \vec{b} and \vec{c} respectively, show that

the length of perpendicular from C on AB is $\frac{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}{|\vec{b} - \vec{a}|}$.

(527) Let $N \times N$ be the set of ordered pairs of natural numbers . Also let R be the relation in $N \times N$, defined by $(a,b)R(c,d) \Leftrightarrow a + d = b + c$.Show that R is an equivalence relation

(528) Let X be non empty set . P(X) be its power set . let * be an binary operation defined on elements of P(X) by , $A * B = A \cup B \forall A, B \in P(X)$ then (i) Prove that * is a binary operation in P(X). (ii) Is * commutative ? (iii)Is associative (iv) Find the identity element of in P(X) w.r.t. * . (v) Find all the invertible of P(x) . (vi) If \otimes is another binary operation defined on P(X) as $A \otimes B = A \cap B$ then verify that \otimes distributive over *

(529) Solve the differential equation: $(1 + y^2)dx = (\tan^{-1} y - x)dy, y(0) = 0$.

(530) Find the distance of the point $(-2,3,- 4)$ from the line $\frac{x + 2}{3} = \frac{2y + 3}{4} = \frac{3z + 4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$.

(531) The top of a ladder 6 meters long is resting against a vertical wall on a level pavement, when the ladder begins to slide outwards. At the moment when the foot of the ladder is 4 meters from the wall, it is sliding away from the wall at the rate of 0.5 m / sec . How fast is the top – sliding downwards at this instance? How far is the foot from the wall when it and the top are moving at the same rate?

(532) Evaluate: $\int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$.

(533) Prove that : $\tan \left[\frac{1}{2} \sin^{-1} \frac{2x}{1+x^2} + \frac{1}{2} \cos^{-1} \frac{1-y^2}{1+y^2} \right] = \frac{x+y}{1-xy}$.

(534) Show that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$.

(535) Find the distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured along a line parallel to $\frac{x + 4}{2} = \frac{y + 6}{3} = \frac{z - 1}{-6}$.

(536) Evaluate: $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$.

(537) Evaluate : $\int \frac{2 \sin 2\phi - \cos \phi}{6 - \cos^2 \phi - 4 \sin \phi} d\phi$.

(538) If $y = \cos(\cos x)$;Prove that $\frac{d^2y}{dx^2} - \cot x \frac{dy}{dx} + y \sin^2 x = 0$.

(539) Given that $\cos \frac{x}{2} \cdot \cos \frac{x}{4} \cdot \cos \frac{x}{8} \dots = \frac{\sin x}{x}$, prove that

$$\frac{1}{2^2} \sec^2 \frac{x}{2} + \frac{1}{2^4} \sec^2 \frac{x}{4} + \dots = \cos^2 x - \frac{1}{x^2}$$

(540) If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, prove that $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$.

(541) Given $\vec{a} = 3\hat{i} - \hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$, express \vec{b} as $\vec{b}_1 + \vec{b}_2$ where \vec{b}_1 is parallel to \vec{a} & \vec{b}_2 is perpendicular to \vec{a} .

(542) Using Lagrange's mean value theorem, find a point on the curve $y = \sqrt{x-2}$ defined on the interval $[2, 3]$, where the tangent is parallel to the chord joining the end points of the curve.

(543) Prove that:
$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3.$$

(544) If \vec{a} and \vec{b} are unit vectors and θ is the angle between them, then show that $\sin \frac{\theta}{2} = \frac{1}{2} |\vec{a} - \vec{b}|$.

(545) Evaluate:
$$\int \frac{\sqrt{x^2+1}[\log(x^2+1) - 2\log x]}{x^4} dx.$$

(546) Evaluate:
$$\int_0^1 \frac{1-x^2}{x^4+x^2+1} dx.$$

(547) If $y = f\left(\frac{2x-1}{x^2+1}\right)$ and $f'(x) = \sin x^2$, find $\frac{dy}{dx}$.

(548) Using differential find the approximate value of $\sqrt{5}$.

(549) Find the area of the region bounded by the curve $y^2 = 4a^2(x-1)$ and the lines $x=1$ & $y=4a$.

(550) Prove that $f(x) = |x+2|$ is continuous at every $x \in R$ but fail to differentiable at $x = -2$.

(551) Using integration, find the area of the two parabolas $4y^2 = 9x$ & $3x^2 = 16y$. Also find the angle between two curves.

(552) Evaluate:
$$\int x(\tan^{-1} x)^2 dx.$$

(553) The sum of the mean and variance of a Binomials distribution is 15 and the sum of their squares is 117. Determine the distribution.

(554) Using the properties of determinants, show that
$$\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = (a^3 - 1)^2.$$

(555) Solve the initial value problems: $\sqrt{1-y^2} dx = (\sin^{-1} y - x) dy, y(0) = 0.$

(556) Show that
$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3.$$

(557) Let \oplus be a binary operation on the set of natural numbers N given by $a \oplus b = \text{L.C.M. of } a \text{ and } b$. Find (i) $5 \oplus 7, 20 \oplus 16$ (ii) Is \oplus commutative? (iii) Is \oplus associative? (iv) Find identity element of \oplus in N .

(558) Evaluate: $\int_{-1}^{3/2} |x \sin \pi x| dx$.

(559) Evaluate $\int_0^4 (|x-1| + |x-2| + |x-3|) dx$.

(560) In a particular city, 24% of the families earn less than Rs 3 lac annually, 70% earn less than Rs 8 lac annually. The probability that a family owns a car is 10% if earning is below Rs 3 lac, 55% if earning is between Rs 3 lac and Rs 8 lac, and 90% if earning is above Rs 8 lac. If we know that a family does have a car, what is the probability that its earning is between 3 lac and 8 lac.

(561) Let X be the random variable which assumes values 0,1,2,3 such that $3P(X=0) = 2P(X=1) = P(X=2) = 4P(X=3)$. Find the probability distribution of X. Also find mean and variance.

(562) If $x^p y^q = (x+y)^{p+q}$ Prove that $\frac{dy}{dx} = \frac{y}{x}$.

(563) Evaluate: $\int_0^{\pi/4} \frac{\tan^3 x}{1 + \cos 2x} dx$.

(564) The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ .

(565) If $\vec{A}, \vec{B}, \vec{C}$ are unit vectors, $\vec{A} \cdot \vec{B} = \vec{A} \cdot \vec{C} = 0$ and angle between \vec{B} and \vec{C} is $\frac{\pi}{6}$, then prove that $\vec{A} = \pm 2(\vec{B} \times \vec{C})$.

(566) Evaluate $\int \frac{(x^2 + 1)(x^2 + 2)}{(x^2 + 3)(x^2 + 4)} dx$.

(567) Solve the differential equation $x \frac{dy}{dx} + y = x \cos x + \sin x$ given that $y\left(\frac{\pi}{2}\right) = 1$.

(568) If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, find a vector \vec{c} such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$.

(569) If $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

(570) If $y = 2 \tan^{-1}\left(\frac{x\sqrt{2}}{1-x^2}\right) + \log\left(\frac{1+x\sqrt{2+x^2}}{1-x\sqrt{2+x^2}}\right)$, show that $\frac{dy}{dx} = \frac{4\sqrt{2}}{1+x^4}$.

(571) Use differentials to find the approximate value of $\sqrt{0.037}$.

(572) In a competitive examination, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that the answer is correct, given that he copied it is $\frac{1}{8}$. Find the probability that he knows the answer to the question, given that he correctly answered it.

(573) Solve the differential equation $(x^2 + 1)\frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$.

(574) Let $f : \{2, 3, 4, 5\} \rightarrow \{3, 4, 5, 9\}$ and $g : \{3, 4, 5, 9\} \rightarrow \{7, 11, 15\}$ be functions defined as $f(2) = 3, f(3) = 4, f(4) = f(5) = 5$ and $g(3) = g(4) = 7$ and $g(5) = g(9) = 11$. Find gof .

(575) Show that the points A, B, C and D with position vectors $\vec{a}, \vec{b}, \vec{c}$ and \vec{d} respectively such that $2\vec{a} + 3\vec{b} - \vec{c} - 4\vec{d} = \vec{0}$ are coplanar.

(576) If $y = \sqrt{\frac{1 - \sin 2x}{1 + \sin 2x}}$, show that $\frac{dy}{dx} + \sec^2\left(\frac{\pi}{4} - x\right) = 0$.

(577) Prove that :
$$\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix} = 4(b+c)(c+a)(a+b).$$

(578) Prove that $\int_0^{\pi/2} \log \sin x dx = -\frac{\pi}{2} \log 2$.

(579) Evaluate: $\int_0^{\pi/4} \log(1 + \tan x) dx$.

(580) The probabilities of A, B, C solving a problem are $1/3, 2/7$ and $3/8$ respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them can solve it.

(581) Let $f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2}, & \text{if } x < 0 \\ a, & \text{if } x = 0 \\ \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4}, & \text{if } x > 0 \end{cases}$ Determine the value of a so that $f(x)$ is continuous at $x = 0$.

(582) Let $N \times N$ be the set of ordered pairs of natural numbers. Also let R be the relation in $N \times N$, defined by $(a, b)R(c, d) \Leftrightarrow a + d = b + c$. Show that R is an equivalence relation.

(583) let $*$ be an binary operation defined $a * b = a + b + ab$ on the set $R - \{-1\}$, then (i) Prove that $*$ is a binary operation $R - \{-1\}$ (ii) Is $*$ commutative? (iii) Is associative (iv) Find the identity element $R - \{-1\}$ w.r.t. $*$. and also prove that every element of $R - \{-1\}$ is invertible.

(584) If $\cos^{-1} p + \cos^{-1} q + \cos^{-1} r = \pi$, then prove that $p^2 + q^2 + r^2 + 2pqr = 1$.

(585) Find the distance of the point $(3, 4, 5)$ from the plane $x + y + z = 2$ measured parallel to the line $2x = y = z$.

(586) Find the vector and Cartesian equation of the plane containing the two parallel lines $\frac{x - 4}{1} = \frac{y - 3}{-4} = \frac{z - 2}{5}$ & $\frac{x - 3}{1} = \frac{y + 2}{-4} = \frac{z}{5}$. Also find the inclination of this plane with the XY plane.

(587) If $y = \sin(\sin x)$, prove that $\frac{d^2y}{dx^2} + \tan x \cdot \frac{dy}{dx} + y \cos^2 x = 0$.

(588) If $y = a^{x^a x^{a^{\dots^{\infty}}}}$ prove that $\frac{dy}{dx} = \frac{y^2 \log y}{x(1 - y \log x \cdot \log y)}$.

(589) If $x = 2 \sin^{-1} \sqrt{\frac{y}{2} - \sqrt{2y - y^2}}$, prove that $\frac{dy}{dx} = \sqrt{\frac{2-y}{y}}$.

(590) If $x \neq y \neq z$ and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then prove that $xyz = -1$.

(591) Solve the differential equation, $\frac{dy}{dx} + y \sec^2 x = \tan x \sec^2 x$; $y(0) = 1$.

(592) Using limits of sum find the integral of $\int_1^3 e^{2x} dx$.

(593) Solve the differential equation: $x \cos \frac{y}{x} (y dx + x dy) = y \sin \frac{y}{x} (x dy - y dx)$.

(594) Show that $Ax^2 + By^2 = 1$ is a solution of the differential equation $x \left[y \left(\frac{d^2 y}{dx^2} \right) + \left(\frac{dy}{dx} \right)^2 \right] = y \left(\frac{dy}{dx} \right)$.

(595) Evaluate: $\int_{-a}^a \sqrt{\frac{a-x}{a+x}} dx$.

(596) Show that $\begin{vmatrix} b^2 + c^2 & ab & ac \\ ba & c^2 + a^2 & bc \\ ca & cb & a^2 + b^2 \end{vmatrix} = 4a^2 b^2 c^2$.

(597) If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2 y}{dx^2} - (m+n) \frac{dy}{dx} + mny = 0$.

(598) Show that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$.

(599) If $x = \tan \left(\frac{1}{a} \log y \right)$, show that $(1+x^2) \frac{d^2 y}{dx^2} + (2x-a) \frac{dy}{dx} = 0$.

(600) Find the value of $2 \tan^{-1} \left(\frac{1}{5} \right) + \sec^{-1} \left(\frac{5\sqrt{2}}{7} \right) + 2 \tan^{-1} \frac{1}{8}$.

(601) Using vectors prove that in a triangle ABC, $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$, a, b and c are sides opposite to A, B and C respectively.

(602) If $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ & $\vec{b} = 3\hat{i} + \hat{j} + 2\hat{k}$, find a unit vector which is linear combination of \vec{a} & \vec{b} and is also perpendicular to \vec{a} .

(603) Evaluate: $\frac{dy}{dx}$ if $y = \frac{x}{2}\sqrt{x^2 - 16} - 8\log|x + \sqrt{x^2 - 16}| + C$.

(604) Find the equation of the plane passing through the point $2\hat{i} - \hat{k}$ and parallel to the lines

$$\frac{x}{-3} = \frac{y - 2}{4} = z + 1 \quad \text{and} \quad \frac{x - 4}{2} = \frac{1 - y}{2} = 2z.$$

(605) Evaluate $\int \frac{\sin^2 x + \cos^2 x}{\cos^2 x + \sin^4 x} dx$.

(606) If $(x - a)^2 + (y - b)^2 = c^2$, for some $c > 0$, prove that $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} \frac{d^2y}{dx^2}$ is a constant independent of a and b.

(607) If a, b, c are all positive and distinct, show that $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ has a negative value.

(608) An air force plane is ascending vertically at the rate of 100 km/h. If the radius of the earth is r km, how fast is the area of the earth, visible from the plane, increasing at 3 minutes after it started ascending? Given that the visible area A at height h is given by

$$A = \frac{2\pi r^2 h}{r + h}.$$

(609) Evaluate: $\int_{-1}^1 \{x + [x]\} dx$.

(610) Three dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

(611) Water is running into a conical vessel 12 cm deep and 4 cm in radius at the rate of 0.2 cu cm/s. When the water is 6 cm deep, find at what rate is (i) The water is rising? (ii) The water surface area increasing?

(612) A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi vertical angle is $\tan^{-1}(1/2)$. Water is poured into it at a constant rate of 5 cubic meter per minute. Find the rate at which the level of the water is rising at the instant when the depth of the water in the tank is 10 cm.

(613) Evaluate: $\int \frac{x^2 + 4}{x^4 + x^2 + 16} dx$.

(614) Determine whether or not the following pairs of lines intersect. If these intersect, find the point of intersection, otherwise obtain the shortest distance between them:
 $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(3\hat{i} - \hat{j})$
 $\vec{r} = 4\hat{i} - \hat{k} + \mu(2\hat{i} + 3\hat{k})$

- (615) State when the line $\vec{r} = \vec{a} + \lambda \vec{b}$ is a parallel to the plane $\vec{r} \cdot \vec{n} = d$. Show that the line $\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} + \hat{j} + 4\hat{k})$ is parallel to the plane $\vec{r} \cdot (-2\hat{i} + \hat{k}) = 5$. Also find the distance between the line and the plane.
- (616) Find the planes bisecting the angles between the planes $x + 2y + 2z = 9$, $4x - 3y + 12z + 13 = 0$ and points out which plane bisects the acute angle.
- (617) In a regular hexagon ABCDEF, if $\vec{AB} = \vec{a}$ & $\vec{BC} = \vec{b}$ then express $\vec{CD}, \vec{DE}, \vec{EF}, \vec{FA}, \vec{AC}, \vec{AD}, \vec{AE}$ & \vec{CE} in term of a and b.
- (618) Solve the differential equation $(1 - x^2) \frac{dy}{dx} - xy = x^2$, given $y = 2$ when $x = 0$.
- (619) Find the shortest distance between the following lines. $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$
- (620) Solve the differential equation : $x \frac{dy}{dx} = y(\log y - \log x + 1)$.
- (621) Evaluate: $\int_0^{\pi} \frac{xdx}{1 + \cos^2 x}$
- (622) Find the equation of the plane parallel to line $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ and containing the lines $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ in vector and Cartesian form, also find distance of plane from origin.
- (623) If $f(x)$ is a continuous function defined on $[0, 2a]$, then $\int_0^{2a} f(x) dx = \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f(2a-x) = f(x) \\ 0, & \text{if } f(2a-x) = -f(x) \end{cases}$
- (624) Evaluate: $\int_0^{\pi/2} \frac{\sin x + x}{1 + \cos x} dx$.
- (625) If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, prove that $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$.
- (626) If $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x} & \text{if } x \neq \frac{\pi}{2} \\ 3 & \text{if } x = \frac{\pi}{2} \end{cases}$ find the value of k if f is continuous at $x = \frac{\pi}{2}$.
- (627) If $x = a(\theta - \sin \theta)$, $y = a(1 + \cos \theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$.
- (628) Two bags A and B contain 4 white 3 black balls and 2 white and 2 black balls respectively. From bag A two balls are transferred to bag B. Find the probability of drawing (a) 2 white balls from bag B? (b) 2 black balls from bag B? (c) 1 white and 1 black balls from bag B.

- (629) Find the equations of tangent lines to the curve $y = 4x^3 - 3x + 5$ which are perpendicular to the line $9y + x + 3 = 0$.
- (630) A candidate has to reach the examination centre in time. Probability of him going by bus or scooter or by other means of transport are $3/10$, $1/10$, $3/5$ respectively. The probability that he will be late is $1/4$ and $1/3$ respectively, if he travel by bus or scooter but he reaches in time if he uses any other mode of transport. He reached late at the center. Find the probability that he traveled by bus.
- (631) Show that the function $f(x) = |x+2|$ is a continuous at every $x \in R$ but fails to be differentiable at $x = -2$.
- (632) The probability that an event happens in one trial of an experiment is $.3$. Three independent trials of the experiment are performed. Find the probability that the event happens at least once.
- (633) Evaluate : $\int_{-\pi}^{\pi} \frac{2x(1 + \sin x)dx}{1 + \cos^2 x}$.
- (634) Show that the points A,B,C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ respectively, are the vertices of a right triangle. Also find the remaining angles of the triangle.
- (635) A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12m, find the dimensions of the rectangle that will produce the largest area of the window.
- (636) Find the equation of the plane passing through the intersection of the planes, $2x + 3y - z + 1 = 0$; $x + y - 2z + 3 = 0$ and perpendicular the plane $3x - y - 2z - 4 = 0$. Also find the inclination of this plane with the xy plane.
- (637) Water is dripping out from a conical funnel of semi vertical angle $\pi/4$ at the uniform rate of 2 sq.cm /sec in its surface area through a tiny hole at the vertices in the bottom. When the slant height of the water is 4 cm, find the rate of decrease of the slant height of the water.
- (638) Evaluate : $\int_0^{\pi} \frac{x dx}{1 + \cos \alpha \sin x}$

PART – C

- (639) Show that the matrix, $A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$ satisfies the equation, $A^3 - A^2 - 3A - I_3 = O$. Hence, find A^{-1} .

- (640) A manufacturer makes two type of toys A and B. Three machines are needed for this purpose and the time (in minutes) required for each toy on the machine is given below.

Types of Toys	Machines		
	I	II	III

A	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each toy of type A is Rs. 7.50 and that on each toy of type B is Rs. 5, find how many of each type should be manufactured in a day to get maximum profit .

(641) Show that a right circular cylinder, which is open at the top and has a given surface area, will have the greatest volume if its height is equal to the radius of its base.

(642) If $A = \begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{pmatrix}$, find A^{-1} and hence solve the following system of equation :

$$2x + y + 3z = 3 ; 4x - y = 3 ; -7x + 2y + z = 0 .$$

(643) Colored balls are distributed in four boxes as shown in the following table:

Box	Color			
	Black	White	Red	Blue
I	3	4	5	6
II	2	2	2	2
III	1	2	3	1
IV	4	3	1	5

A box is selected at random and then a ball is randomly drawn from the selected box. The color of the ball is black. Find the probability that ball drawn is from box III.

(644) If the length of three sides of a trapezium, other than the base are equal to 10cm each, then find the area of trapezium when it is maximum.

(645)]A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2m and volume is 8m³. If building of tank cost Rs. 70 per square meters for the base and Rs. 45 per square meters for sides. What is the cost of least expensive tank?

(646) Find the area of the region : $\{(x, y) : y^2 \geq 6x, x^2 + y^2 \leq 16\}$.

(647) Using elementary transformation, find the inverse of the matrix : $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix}$.

(648) Draw the rough sketch of the region enclosed between the circles $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 1$. Using integration, find the area of the enclosed region.

(649) Evaluate : $\int_1^2 (x^2 + x + 2) dx$ as a limit of sums .

(650) Evaluate : $\int_0^1 \sin^{-1}(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}) dx, 0 \leq x \leq 1$

(651) Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point (1,3,4) from the plane $2x-y+z+3=0$. find also, image of the point in the plane.

- (652) An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 400 is made on each ist class ticket and a profit of Rs. 600 is made on each economy class ticket. The airline reserves at least 20 seats for the executive class. however, at least 4 times as many passengers prefer to travel by economy class, then by the executive class. Determine how many tickets of each type must be sold, in order to maximize profit for the airline. What is the maximum profit? Make an L.P.P. and solve it graphically.
- (653) A fair die is rolled. If 1 turns up, a ball is picked up at random from bag A, if 2 or 3 turns up, a ball is picked up at random from bag B, otherwise a ball is picked up from bag C. Bag A contains 3 red and 2 white balls, bag B contains 3 red and 4 white balls and bag C contains 4 red and 5 white balls. The die is rolled, a bag is picked up and a ball is drawn from it. If the ball drawn is red, what is the probability that bag B was picked up?
- (654) An amount of Rs.5000 is put into the three investment at the rate of interest of 6 % , 7 % and 8% per annum respectively . The total annual income is Rs.358 .If the combined income from the first two investment is Rs.70 more than the income from the third , find the amount of each investment by matrix method .
- (655) Find the area of the region $\{(x, y) : y^2 \leq 8x, x^2 + y^2 \leq 9\}$.
- (656) Evaluate: $\int_a^b x dx$ as limit of a sum.
- (657) A point on the hypotenuse of right triangle is at a distance a and b from the sides of a triangle . Show that the minimum length of the hypotenuse is $[a^{2/3} + b^{2/3}]^{3/2}$.
- (658) Form a differential equation of the curve $xy = Ae^x + Be^{-x} + x^2$, A and B are arbitrary constants.
- (659) Evaluate : $\int \frac{x^2}{(x \sin x + \cos x)^2} dx$.
- (660) A factory owner purchases two types of machines, A and B for his factory. The requirements and the limitations for the machines are as follows:
- | Machine | Area occupied | Labour force | Daily output(in units) |
|---------|---------------|--------------|------------------------|
| A | $1000m^2$ | 12 men | 60 |
| B | $1200m^2$ | 8 men | 40 |
- He has maximum area of $9000 m^2$ available, and 72 skilled labourers who can operate both the machines. How many machines of each type should he buy to maximize the daily output?
- (661) Find the area of the region bounded by the curve $y^2 = 4a^2(x-1)$ and the lines $x = 1$ & $y = 4a$.
- (662) A doll manufacturing company manufactures two types of dolls , type A and type B .Each doll of type A takes twice as long as to produce a doll of type B . The company has enough time to manufacture a maximum of 2000 dolls per day if it produces only type B dolls .The supply of plastic is sufficient enough to manufacture

a total of 1500 dolls every day .The type A doll requires fancy dress of which only 550 are available daily .If the company makes a profit of Rs8 on each type of doll A and Rs 5 each type of doll B , how many of each of types of dolls should be manufactured everyday so as to get a maximum profit ? Also find the maximum profit .

(663) A cottage industry manufactures pedestal lamps and wooden shades , each requiring the use of grinding cutting machine and a sprayer .It takes 2 hours on a grinding/cutting machine and 3 hours on a sprayer to manufacture a pedestal lamp. It takes 1 hour on a grinding/cutting machine and 2 hours on a sprayer to manufacture a shade .On any day , the sprayer is available for at the most 20 hours and the grinding/cutting

machine for at the most 12 hours .The profit from the sale of lamp is Rs.5 and that from the shade is Rs.3 Assuming that he manufacture can sell all the lamps and shades that he produces . How should he schedule his daily production in order to maximize his profit .

(664) An open box with a square base is to be made out of a given quantity of card board of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

(665) Let the number of times a candidate applies for a job be X and $P(X=x)$ denotes the probability that he will be selected x times. Given that

$$P(X = x) = \begin{cases} (k+1)x & , \text{if } x = 0 \\ 2kx & , \text{if } x = 1 \text{ or } 2 \\ k(6-x), & \text{if } x = 3 \text{ or } 4 \text{ or } 5 \end{cases} \quad \text{where } k \text{ is a +ve real number.}$$

(a) Find the value of k.(b) What is the probability that he will be selected exactly three times.(c) What is the probability that he will be selected at least once.(d) Find the mean and variance of the probability distribution of X .

(666) Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi – vertical angle α is $\frac{4}{27} \pi h^3 \tan^2 \alpha$.

(667) Let $A = \begin{bmatrix} 0 & -\tan(\alpha/2) \\ \tan(\alpha/2) & 0 \end{bmatrix}$ and I be the identity matrix of order 2. Show that $I + A = (I - A) \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$.

(668) Show that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

(669) Suppose every gram of wheat provides 0.1 gm of proteins and 0.25 gm of carbohydrates and the corresponding values for rice are 0.05 gm and 0.5 gm respectively. Wheat costs Rs 4 per kg and rice Rs 6. The minimum daily requirements of proteins and carbohydrates for an average child are 50 gm and 200 gm respectively.

In what quantities should wheat and rice be mixed in the daily diet to provide the minimum daily requirements of proteins and carbohydrates, if both the items are to be taken up in each diet

(670) The section of a window consists of a rectangle surmounted by an equilateral triangle. If the perimeter be given as 16 m, find the dimensions of the window in order that the maximum amount of light may be admitted.

(671) Evaluate: $\int_1^4 (2x^2 + 3x + 5) dx$ as limit of a sum.

(672) Find the area of the region included between the parabolas $y = \frac{3x^2}{4}$ and the line $3x - 2y + 12 = 0$.

(673) Using integration find the area bounded by the lines $x + 2y = 2$, $y - x = 1$ and $2x + y = 7$.

(674) Three bad eggs got mixed up with seven good eggs. If three eggs are drawn (without replacement) from 10 eggs. Find the mean and variance for the number of bad eggs drawn.

(675) Find the area of the region $\{(x, y): y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$.

(676) Show that each of the relation R in the set $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$, given by

(i) $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$

(ii) $R = \{(a, b) : a = b\}$ is an equivalence relation. Find the set of all elements to 1 in each cases.

(677) Find the foot & image of the point (0,2,3) on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. Also find the equation of plane contain the line and passes through (0, 2, 3).

(678) Let X be non empty set. P(X) be its power set. let * be an binary operation defined on elements of P(X) by, $A * B = A \cap B \forall A, B \in P(X)$ then (i) Prove that * is a binary operation in P(X). (ii) Is * commutative? (iii) Is associative (iv) Find the identity element of in P(X) w.r.t. *. (v) Find all the invertible of P(x). (vi) If \otimes is another binary operation defined on P(X) as $A \otimes B = A \cup B$ then verify that \otimes distributive over *.

(679) Prove that the image of the point (3, -2, 1) in the plane $3x - y + 4z = 2$ lies on the plane $x + y + z + 4 = 0$.

(680) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of linear equations $x + 2y + z = 4$, $-x + y + z = 0$, $x - 3y + z = 2$.

(681) A given quantity of metal is to be cast into a half cylinder with a rectangular base and semi-circular ends. Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its semi-circular ends is $\pi : (\pi + 2)$.

(682) Show that the semi- vertical angle of a right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

(683) Find the vector and Cartesian equation of the plane containing the two lines $\vec{r} = 2i + j - 3k + \lambda(i + 2j + 5k)$ & $\vec{r} = 2i + j - 3k + \mu(3i - 2j + 5k)$.

Also find the inclination of this plane with the XZ plane .

(684) Prove that the function $f:R_+ \rightarrow]-5,\infty)$ given by $f(x)=9x^2+6x-5$ is invertible. Find the inverse of f .

(685) Sketch the graph $f(x) = \begin{cases} |x-2|+2 & x \leq 2 \\ x^2-2 & x > 2 \end{cases}$. Evaluate $\int_0^4 f(x)dx$. What does this value represent on the graph?

(686) Find the area bounded by the curves $y = 6x - x^2$ & $y = x^2 - 2x$.

(687) A furniture firm manufactures chairs and tables, each requiring the use of three machines A,B and C. Production of one chair requires 2 hours on machine A, 1 hour on machine B and 1 hour on machine C. Each tables requires 1 hour each on machine A and B and 3 hours on machine C. The Profit earned by selling one chair is Rs . 30 while by selling one table the profit is Rs 60.The total time available per week on machine A is 70 hours, on machine B is 40 hours and on machine C is 90 hours. How many chairs and tables should be made per week so as to maximize profit ? Formulate the problem as L.P.P and solve it graphically.

(688) Find the area of the region $\{(x, y) : y^2 \leq 8x, x^2 + y^2 \leq 9\}$.

(689) Evaluate: $\int_0^{\pi/2} \sin 2x \tan^{-1}(\sin x) dx$.

(690) A housewife wishes to mix together two kinds of food F_1 & F_2 in such a way that the mixture contains atleast 10 units of vitamin A , 12 units of vitamin B and 8 units of vitamin C . The vitamin contents of one kg of foods F_1 & F_2 are as below :

	Vitamina	VitaminB	VitaminC
Food F_1	1	2	3
Food F_2	2	2	1

One kg of food F_1 costs Rs 6 and one kg of food F_2 costs Rs 10. Formulate the above problem as a linear programming problem, and use iso – cost method to find the least cost of the mixture which will produce the diet .

(691) Two godowns A and B have a grain capacity of 100 quintals and 50 quintals respectively. They supply to three shops D, E and F, whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from godowns to

Transportation cost per quintal (in Rs.)		
From To	A	B
D	6.00	4.00
E	3.00	2.00
F	2.50	3.00

the shops are given below : How should the supplies be transported in order that transportation cost is minimum ?

(692) Define the line of shortest distance between two skew lines .Find the magnitude and the equation of the line of the shortest distance between the following lines :

$$\frac{x}{2} = \frac{y}{-3} = \frac{z}{1} \quad \text{and} \quad \frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2} .$$

(693) Find the dimensions of the rectangle of perimeter 36cm which will sweep out a volume as large as possible when revolve about one of its sides. Also find the maximum volume.

(694) Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of

equations : $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

(695) A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top, by cutting off squares from each corners and folding up the flaps. What should be the side of the square to be cut off so that the volume of the boxes is maximum possible?

(696) If the sum of the length of the hypotenuse and a side of a right angled triangle is given , show that the area of the triangle is maximum when the angle between them is $\pi/3$.

(697) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ & $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Find also the point of intersection of these lines .Also find the equation of plane contain them .

(698) let $A = N \times N$ and $*$ be an binary operation defined by $(a, b) * (c, d) = (ac, bd) \forall a, b, c, d \in N$ on the set R , then (i) Prove that $*$ is a binary operation on N (ii) Is $*$ commutative ? (iii) Is associative (iv) Find the identity element for $*$ on $N \times N$ if any .

(699) Let $A = \{2, 3, 4, 5, 6, 7, 8, 9\}$.Let R be the relation on A defined by $\{(x, y) : x \in A \ \& \ x \text{ divides } y\}$. Find (i) R (ii) Domain of R (iii) Range of R (iv) R^{-1} state whether or not R^{-1} , is (a) Reflexive (b) symmetric (c) transitive

(700) If a young man rides his motor cycle at 25 km per hour , he has to spend Rs 2 per kilometre on petrol; if he rides at a faster speed of 40 km per hour, the petrol cost increases to Rs 5 per kilometre . He has Rs 100 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Express this as a linear programming problem and then solve it .

(701) Find the area of the region $\{(x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$.

- (702) The sum of the surface areas of a rectangular parallelepiped with sides x , $2x$ and $x/3$ and a sphere of radius r is given to be constant. Prove that sum of their volume is minimum if $x = 3r$. Also find the minimum value of the sum of the volumes.
- (703) Find the shortest distance of the point $(0, c)$ from the parabola $y = x^2$, where $0 \leq c \leq 5$.
- (704) Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.
- (705) A retired person has Rs. 70,000 to invest and two types of bonds are available in the market for investment. First type of bond yields an annual income of 8% on the amount investment and the second type of bonds yields 10% per annum. As per norms, he has to invest a minimum of Rs. 10,000 in the first type and not more than Rs. 30,000 in second type. How should he plan his investment so as to get maximum return, after one year of investment?
- (706) Find A^{-1} , if $A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$. Hence solve the following system of linear equations: $x + 2y + 5z = 10$, $x - y - z = -2$, $2x + 3y - z = -11$.
- (707) Prove that a conical tent of given capacity will require the least amount of canvas when the height is $\sqrt{2}$ times the radius of the base.
- (708) Using integration, compute the area of triangle with vertices $(1, 1)$; $(3, -1)$ $(5, 4)$.
- (709) The sum of the surface areas of a sphere and a cube is given. Show that when the sum of their volumes is least, the diameter of the sphere is equal to the edge of the cube.
- (710) A variable plane is at a constant distance p from the origin and meets the axes in points A, B, C . Through A, B, C planes are drawn parallel to the coordinate planes. Prove that the locus of their point of intersection is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.
- (711) A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs 8000 on each piece of model A and Rs 12000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realise a maximum profit? What is the maximum profit per week?
- (712) Find the area of the region included between the parabolas $y^2 = 4ax$ & $x^2 = 4ay$. where $a > 0$.
- (713) Find the ratio of the areas into which curve $y^2 = 6x$ divides the region bounded by $x^2 + y^2 = 16$.

(714) Evaluate: $\int \frac{e^{\tan^{-1} x}}{(1+x^2)^2} dx$.

(715) Find the matrix P satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

(716) Find the area of the region $\{(x, y): x^2 + y^2 \leq 2ax, y^2 \geq ax, x \geq 0, y \geq 0\}$.

(717) A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12m, find the dimensions of the rectangle that will produce the largest area of the window.

(718) Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6r\sqrt{3}$.

(719) Let a pair of dice be thrown and the random variable X be the sum of the numbers that appears on the two dice. Find the mean or expectation of X.

(720) Find the distance between the point P (6,5,9) and the plane determined by points A(3,-1,2) ,B (5,2,4) and C (-1,-1,6) .

(721) Find the area of the smaller region common to the circle $x^2 + y^2 = 16$ and the parabola $x^2 = 6y$.

(722) There are three urn having the following compositions of black and white balls : urn 1 contain 7 white and 3 black balls , urn 2 contain 4 white and 6 black balls & urn 3 contain 2 white and 8 black balls. One of these urns is chosen with probabilities 0.2, 0.6 and 0.2 respectively. From the chosen urn, two balls are drawn at random without replacement. Both the balls happen to be white. Calculate the probability that the balls drawn were from urn III.

(723) A dietician mixes together two kinds of food in such a way that the mixture contains at least 6 units of vitamin A, 7 unit of vitamin B, 11 units of vitamin C and 9 units of vitamin D. The vitamin contents of 1 kg of food X and 1 kg of food Y are given below : One kg of food X costs Rs. 5, whereas one kg of food Y costs Rs. 8. Find the least cost of the mixture which will produce the desired diet.

	Vitamin A	Vitamin B	Vitamin C	Vitamin D
Food X	1	1	1	2
Food Y	2	1	3	1

(724) A furniture firm manufactures chairs and tables, each requiring the use of three machines A,B and C. Production of one chair requires 2 hours on machine A, 1 hour on machine B and 1 hour on machine C. Each tables requires 1 hour each on machine A and B and 3 hours on machine C. The Profit earned by selling one chair is Rs . 30 while by selling one table the profit is Rs 60.The total time available per week on machine A is 70 hours, on machine B is 40 hours and on machine C is 90 hours. How many chairs and tables should be made per week so as to maximize profit ? Formulate the problem as L.P.P and solve it graphically.

(725) Draw a rough sketch of the curves $y = \sin x$ & $y = \cos x$ as x varies from 0 to $\frac{\pi}{2}$ and find

the area of the region enclosed by them and (i) x – axis (ii) Y- axis.

(726) Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that(i) the youngest is a girl, (ii) at least one is a girl?

(727) There are three urns . Urn I contains 1 white, 2 black and 3 red balls . Urn II contains 2 white, 1 black and 1 red ball. Urn III contain 4 white, 5 black and 3 red balls. One urn is chosen at random and two balls drawn at random . They happen to be white and red . What is the probability that they are from urn I , II or III ?

(728) Show that $\int_0^a \sin^{-1} \sqrt{\frac{x}{a+x}} dx = \frac{a}{2}(\pi - 2)$.

(729) In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

(730) If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ prove that $(aI + bA)^n = a^n I + na^{n-1}bA$, where I is the unit matrix of order 2 and n is a positive integer .

(731) Find the area of the region $\{(x, y) : x^2 \leq y \leq |x|\}$.

(732) A rectangular sheet of paper for a poster is 15000 sq. cm. in area. The margins at the top and bottom are to be 6 cm. wide and at the sides 4 cm. wide. Find the dimensions of the sheet to maximize the printed area.

(733) Using properties of determinants, show that $\Delta = \begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix}$
 $= 2abc(a+b+c)^3$.

(734) Water is running into a conical vessel 12 cm deep and 4 cm in radius at the rate of 0.2 cu cm/s. When the water is 6 cm deep, find at what rate is (i) The water is rising? (ii) The water surface area increasing?

(735) The sum of the perimeter of a circle and a square is k, where k is some constant .prove that the sum of their areas is least when the side of square is double the radius of the circle.

(736) A helicopter is flying along the curve $y = x^2 + 2$. A soldier is placed at the point (3, 2). Find the nearest distance between the soldier and the helicopter.

(737) Evaluate : $\int \frac{1}{\sin x(5 - 4 \cos x)} dx$.

(738) Evaluate : $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$

(739) Using integration, find the area of the region $\{(x, y): |x-1| \leq y \leq \sqrt{5-x^2}\}$

(740) Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar. Also find the equation of the plane contain them .

(741) From a pack of 52 cards, a card is lost. From the remaining 51 cards, two cards are drawn at random (without replacement) and are found to be both diamonds .What is the probability that the lost card was a card of heart ?

(742) A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400calories. Two foods X and Y are available at a cost of Rs 4 and Rs 3 per unit respectively. One unit of food X contains 200 units of vitamins, 1 unit of minerals and 40 calories, whereas 1 unit of food Y contains 100 units of vitamins , 2 units of minerals and 40 calories. Find what combination of foods X and Y should be used to have least cost, satisfying the requirements. Make it an LPP and solve it graphically.

(743) A wire of length 36m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle . What should be the lengths of the two pieces, so that the combined area of the square and the circle is minimum ?

(744) Evaluate: $\int \frac{\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}} dx.$

(745) Find the area of the region bounded by the curves $y = x^2 + 2, y = x, x = 0$ & $x = 3$.

(746) Find the vector equation of the line parallel to the line $\frac{x-1}{5} = \frac{3-y}{2} = \frac{z+1}{4}$ and passing through the point $(3 , 0, - 4)$. Also find the distance between two lines .

(747) Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight

line $\frac{x}{a} + \frac{y}{b} = 1.$

(748) Find the area of the region enclosed between the two circles $x^2 + y^2 = 1$ & $(x-1)^2 + y^2 = 1.$

(749) A category agency has two kitchens to prepare food at two places A and B. From these places ‘ Mid-day Meal ‘ is to be supplied to three different schools situated at P, Q, R. The monthly requirements of the schools are respectively 40,40 and 50 food packets. A packets contains lunch for 1000 students. Preparing capacity of kitchens A and B are 60 and 70 packets per month respectively. The transportation cost per packet from the kitchen to schools is given below :

Transportation cost per packet (in rupees)

To	From	
	A	B
P	5	4
Q	4	2
R	3	5

How many packets from each kitchen should be transported to school so that the cost of transportation is minimum ? Also find the minimum cost .

- (750) Find the probability distribution of the number of white balls drawn in a random draw of 3 balls without replacement from a bag contain 4 white and 6 red balls . Also find the mean and variance of the distribution .
- (751) Find the vector equation in scalar product form of the plane $\vec{r} = \hat{i} - \hat{j} + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k})$.
- (752) Using matrices, solve the following system of equations :
 $\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 4; \frac{2}{x} + \frac{1}{y} - \frac{3}{z} = 0; \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2x \neq 0, y \neq 0, z \neq 0$
- (753) A biased die is twice as likely to show an even number as an odd number. The die is rolled three times. If occurrence of an even number is considered a success, then write the probability distribution of number of successes. Also find the mean number of successes.
- (754) Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi vertical angle α , is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.
- (755) Show that the normal at any point θ to the curve $x = a \cos \theta + a \theta \sin \theta$ and $y = a \sin \theta - a \theta \cos \theta$ is at constant distance from the origin.
- (756) A firm manufactures two types of products A and B and shells them at a profit to Rs. 5 per unit of type A and Rs.3 per unit of type B. Each product is processed on two machines M_1 and M_2 . One unit of type A requires one minute of processing time on M_1 and two minutes of processing time on M_2 ; whereas one unit of type B require one minute of processing time on M_1 and one minute of M_2 . Machines M_1 and M_2 are respectively available for at most 5 hours and 6 hours in a day. Find out how many units of each type of product should firm produce a day in order to maximize the profit. Solve the problem graphically.
- (757) A furniture firm manufactures chairs and tables, each requiring the use of three machines A, B and C. Production of one chair requires 2 hours on machine A, 1 hour on machine B and 1 hour on machine C. Each table requires 1 hour each on machine A and B and 3 hours on machine C. The profit obtained by selling one chair in Rs. 30 while by selling one table the profit is Rs. 60. The total time available per week on

machine A is 70 hours, on machine B is 40 hours and on machine C is 90 hours. How many chairs and tables should be made per week so as to maximize profit? Formulate the problem as L.P.P. and solve it graphically

(758) Find the area of the region : $\{(x, y) : 0 \leq y \leq x^2, 0 \leq y \leq x + 2; 0 \leq x \leq 3\}$.

(759) Find the particular solution of the differential equation

$$(x dy - y dx) y \cdot \sin\left(\frac{y}{x}\right) = (y dx + x dy) x \cos\frac{y}{x}, \text{ given that } y = \pi \text{ when } x=3.$$

(760) From a pack of 52 cards, a card is lost. From the remaining 51 cards, two cards are drawn at random (without replacement) and are found to be both red cards. What is the probability that the lost card was a card of red color?

(761) Find the equation of the plane passing through the points (1,1,1) and containing the line $\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(3\hat{i} - \hat{j} + 5\hat{k})$. Also, show that the plane contains the line $\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \lambda(\hat{i} - 2\hat{j} - 5\hat{k})$.

(762) Two bag A and B contains 4 white and 3 black balls and 2 white and 2 black balls respectively. From bag A, two balls are drawn at random and then transferred to bag B. A ball is then drawn from bag B and is found to be a black ball. What is the probability that the transferred balls were 1 white and 1 black?

(763) In an examination, 10 questions of true- false type are asked. A student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers true and if it falls tails, he answers false. Show that the probability that he answers at most 7 questions correctly is $\frac{121}{128}$.

(764) A toy company manufactures two types of dolls, A & B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for doll of type A. Further the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs. 15 per doll respectively on doll A and B, how many each should be produced weekly in order to maximum profit?

(765) Using matrices, solve the following system of equations:

$$x + \frac{2}{y} + 3xz = -1; 2x - \frac{4}{y} - 3xz = 3; 3x + \frac{6}{y} - 2xz = 4.$$

(766) Draw the rough sketch of the region enclosed between the circles $x^2 + y^2 = 9$ and $(x - 3)^2 + y^2 = 1$. Using integration, find the area of the enclosed region.

Ans $x \geq 0; y \geq 0; x + y \leq 1200; y \leq \frac{x}{2}; x \leq 3y + 600; P = 12x + 16y$
